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GRAVIMETER DESIGN AND OPERATION*

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ABSTRACT

An elementary general discussion of gravimeters and a brief comparison of gravimeter with torsion balance are given. Details as to construction, accuracy, speed of operation and methods of making field observations are given for one type of instrument which is being widely used in geophysical exploration work.

A gravimeter is simply a very precise weighing machine used to find the weight of a certain lump of metal or other material at a series of stations distributed over the area being surveyed. Since the weight of an object is the earth's gravitational attraction on it, this weight will be slightly affected by the nature of subsurface materials and will be slightly larger, for example, at stations where the subsurface material is of higher density or where dense material comes closer to the surface. The changes in weight are so small that the weighing machine must be capable of detecting changes something like one part in ten million.

The final result of a gravimeter survey is a contoured gravity map which shows the value of gravity, that is, the weight of a one gram mass at every point in the region; the same map, incidentally, as that obtained from a torsion balance survey.

The gravimeter and the torsion balance can best be compared by considering the analogous case in which a topographic survey is made by two different possible methods. In the first and usual method the engineer and rodman measure elevations relative to some starting point and can, if a bench mark is available, convert them to sea level elevations. This is the gravimeter method. Gravity differences are measured directly and if a bench mark in the form of a pendulum station is available the relative values can be converted to absolute ones. With the second method, which is never used in practice, the

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engineer lays a straight edge on the ground and measures the angle of slope in a north-south and in an east-west direction and from these data indirectly obtains a topographic map. This is the torsion balance method. One obvious difficulty with the second method is that there might be strictly local surface irregularities which would cause serious errors in the slope measurements, although they would have little effect on the actual elevation. In the same way, local conditions which have negligible effect on a gravimeter may seriously affect a torsion

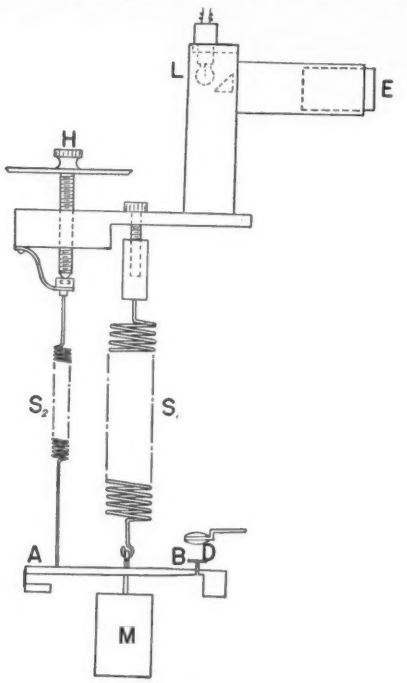


FIG. 1

balance and must be corrected for as, nearly as possible, by means of the so-called "terrain correction." Both instruments, of course, have their advantages. The chief advantages of the gravimeter are speed of operation and freedom from local disturbance. On the other hand, the torsion balance in some cases gives valuable additional information in the form of curvature data.

In a gravimeter, the force of gravity must be opposed by some other kind of force. There are a number of possibilities. Magnetic, electrostatic, or centrifugal forces are interesting to consider although they have not been used so far as I know. The pressure of a gas has

been used but seems to me to require extremely accurate temperature control, about .00003°C., in fact. Use of an elastic force, a spring of some sort, has been more common. The spring material should have a low temperature coefficient and low hysteresis; both steel and quartz have been successfully used. There are many possible mechanical arrangements. The simplest I know of is the Hartley design as shown in Fig. 1. In this instrument the weight of the mass M is supported by the main spring S_1. The cross member AB is hinged at A with a flexible metal strip in such a way that as M moves up and down the end B also moves up and down and causes the small mirror D to tilt from side to side. An image of the filament of the lamp L as re-

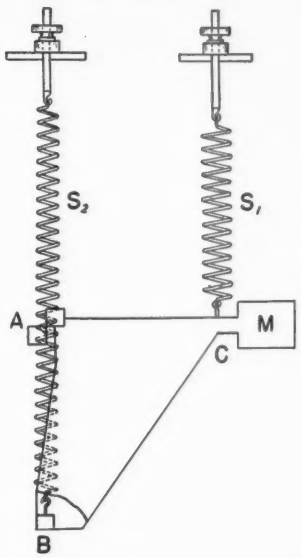


FIG. 2

flected in the mirror D is observed in the eyepiece at E. The tension in the small spring S_2 is adjusted by turning the divided head H until the reflected image of the lamp filament is brought back to a previously chosen reference line in the eyepiece. In other words, variations in the pull of the earth on M are balanced out by changing the pull of the spring S_2, in such a way as to keep the plane of the mirror D always horizontal. Readings are made on the divided head H.

A second and slightly more complicated mechanical arrangement is shown in Fig. 2. This instrument was originally designed in the laboratories of the Humble Oil & Refining Company and is now being built and used by both Humble and Carter in slightly different forms.

A more or less triangular framework ABC is hinged at A by means of flexible metal strips. A mass M is attached to the corner C of the frame work and its weight is supported by the spring S_1. A second spring S_2 is attached to the lower corner B of the framework and pulls directly upward through the hinge line. This spring serves to increase greatly the sensitivity of the instrument. A number of features of the actual instrument have been omitted from this diagram for the sake of clearness. A third very small spring is attached to the frame and works in parallel with S_1. It is used only because when adjustments are necessary it is more convenient to make them on this third small spring than on the heavier main spring S_1.

Means for clamping the moving system and electro-magnetic damping are also provided. There is also a provision for putting on the frame a small rider of known weight used in calibrating the instrument. To read the instrument an optical system is focused from above on a reference line attached to the frame at B and the lateral motions of this reference line corresponding to vertical motions of M are observed.

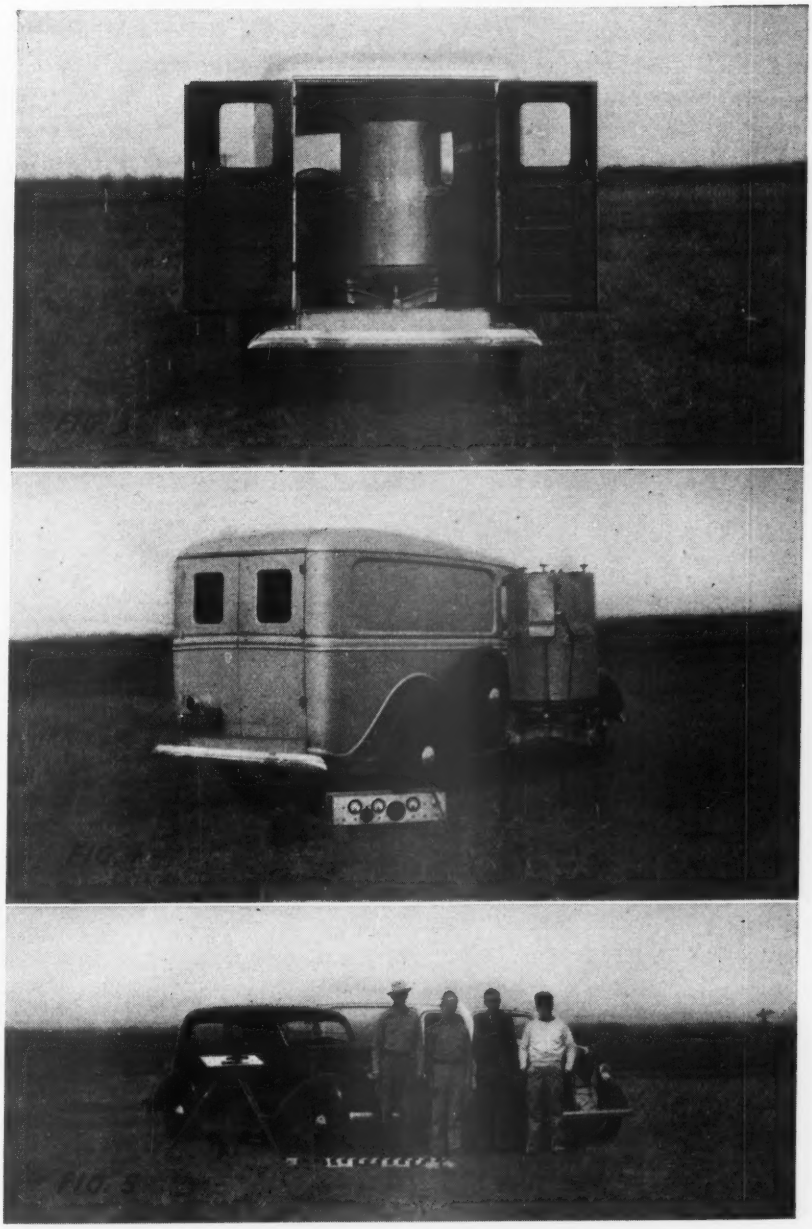
The instrument is mounted in a case, the temperature of which is accurately controlled. The temperature change observed with the present instrument is a slow and constant drift of something like .01°C. per 24-hour day. We have found it desirable to use elevator tests as an additional means of calibrating. The instrument is read on several floors of a 10 or 12 story building and the sensitivity is then computed from the instrument readings and the easily calculated changes in gravity with elevation.

Fig. 3 shows the instrument mounted on its tripod in the truck ready for reading. This tripod goes through holes in the floor of the truck so that vibrations of the truck caused by wind or other disturbances do not interfere with the instrument while it is being read.

Fig. 4 shows another view in which the instrument and accessory apparatus are entirely removed from the truck.

Fig. 5 shows the entire personnel of the party, including the two operators of the instrument and the two surveyors.

Our latest instrument weighs 112 pounds and is readily handled by two men. In making a field survey, readings are usually taken at stations one mile apart and the results are plotted as a drift curve, similar to that shown in Fig. 6. Here it will be noted that in order to establish the drift of the instrument each station has been visited at least twice and some of them have been visited three times during



Gravimeter and Field Equipment

the day. On this particular day the drift was very small so that the drift lines are practically horizontal. The vertical interval between these lines is taken as a measure of the gravity differences between the various stations. A small barometric correction is applied to the readings. This is necessary because changes in the barometric pressure

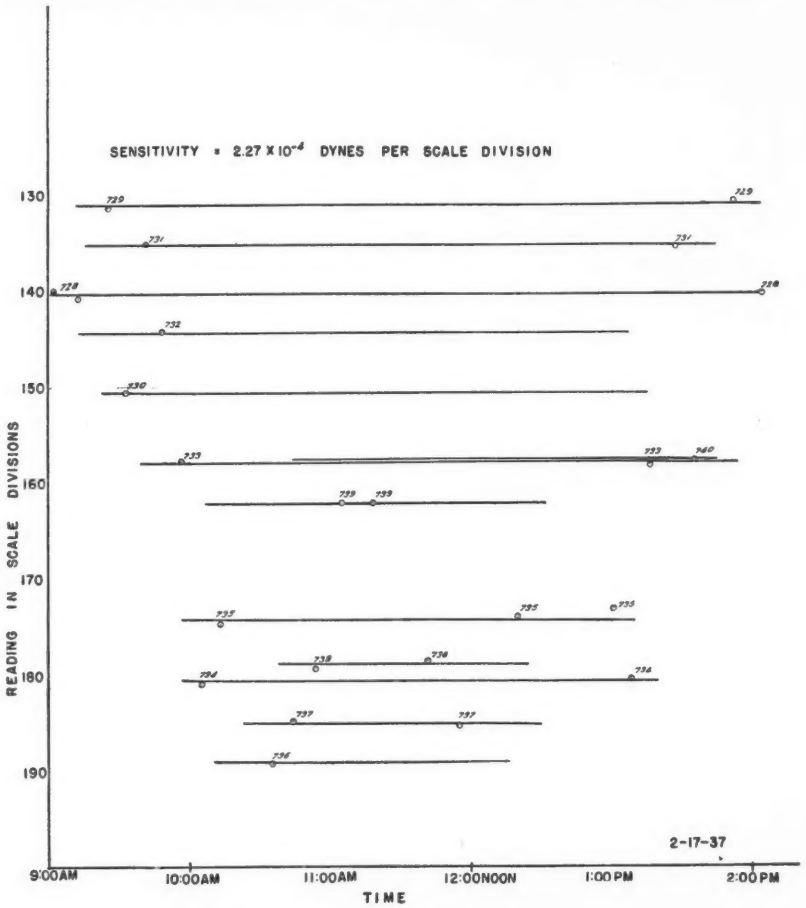


FIG. 6

slightly change the buoyant force of the air on the suspended mass and, consequently, the effective weight of the mass.

During the particular day's work shown by the drift curves in Fig. 6, 24 readings were taken at 13 stations during a period of 5 hours. In other words, the crew takes a reading every 12 minutes driving a mile after each reading and a total of 13 miles of line is

covered in 5 hours. The limiting factor as to the speed of our present operations is the surveying. The gravimeter crew has little difficulty in keeping up with the surveyor and consequently does not work as fast or as many hours per day as it might otherwise.

As to the reliability of the instrument it may be said that we have during the past several months lost about one day per month on

LOOP	TOTAL MILES	TOTAL STATIONS	MAX. GRAVITY DIFFERENCE IN 10^{-4} CGS. UNITS	CLOSURE ERROR 10^{-4} CGS. UNITS
A	18	6	208	2.3
B	20	6	200	2.6
C	33	30	208	1.9
D	50	31	514	4.7
E	22	16	270	1.7

FIG. 7

account of instrument troubles and these troubles have usually been battery failures. An indication of the accuracy of the instrument is given by the data shown in Fig. 7. Five closed loops have been selected at random from some of our field data. Loop A, for example, involved a total of 18 miles of line, 6 gravity stations, a maximum gravity difference of 208 units between the highest and the lowest station, and an error in closing of 2.3 units. Similar information is given on 4 other such loops and it will be seen that the error in closing varies from 1.7 to 4.7 units. The unit used here is 1/10 milli-dyne, or approximately 1/10,000,000 of the total value of gravity.

Before the observed gravity changes are put on the map and con-

toured two corrections are made for the purpose of subtracting gravity changes produced by known and calculable causes and leaving only those due to anomalous subsurface conditions. These are the elevation correction and the latitude correction. The elevation correction is easily calculated except that it involves the density of the surface material and, since this density is not accurately known, this correction must be considered as subject to error and it is desirable to watch for and avoid a direct or inverse correlation between gravity and elevation changes. The latitude correction is made by using one of the well known formulae.

A NEW SOURCE OF DAMPED WAVE TRAINS SUITABLE FOR THE TESTING OF GEOPHYSICAL APPARATUS*

D. G. C. HARE†

ABSTRACT

A newly developed electric circuit is described, by means of which electrical transients of nearly any desired characteristics may be conveniently produced. The circuit is particularly adapted for use with a cathode ray oscilloscope, in that no auxiliary synchronization is necessary. A typical use in the testing of geophysical circuit elements is illustrated, and a novel method of making rough phase-shift measurements is described.

It is generally accepted that the waves which travel through the earth as a result of an explosion such as is used in seismic exploration consist of rather sharply initiated, highly damped trains. These elastic waves are changed to electrical impulses in the receptors, and these impulses then travel through the various networks that form the recorder system or channel. The circuit analysis of such a channel is tedious, at best, when we consider only steady state phenomena; and it is laboriously difficult, if not impossible, for the very transient type of wave we have to deal with in seismic exploration. However, in the routine testing of present equipment and in the development of new apparatus, it is necessary that we know the response of this equipment or apparatus to such waves.

It has been the custom to determine this response by providing a source of damped wave trains, and then examining the character of these wave trains—using, say, a cathode ray oscilloscope—before and after they pass through the apparatus. Thus the effect of the network or apparatus upon the wave form can be determined by comparison.

The most widely used method of producing such waves is the impulse excitation of a circuit containing inductance, capacity, and resistance. By properly choosing the constants of such a circuit, one can get waves of any desired character, but there are certain outstanding drawbacks to this method. For instance, if one is to scan the wave with a cathode ray oscilloscope, some means of synchronizing the sweep circuit of the cathode ray tube and the trip circuit of the wave

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† Rieber Laboratory, Los Angeles, California.

train generator must be provided, in order that there be a steady image on the screen. That this is a discouraging and tricky job will be

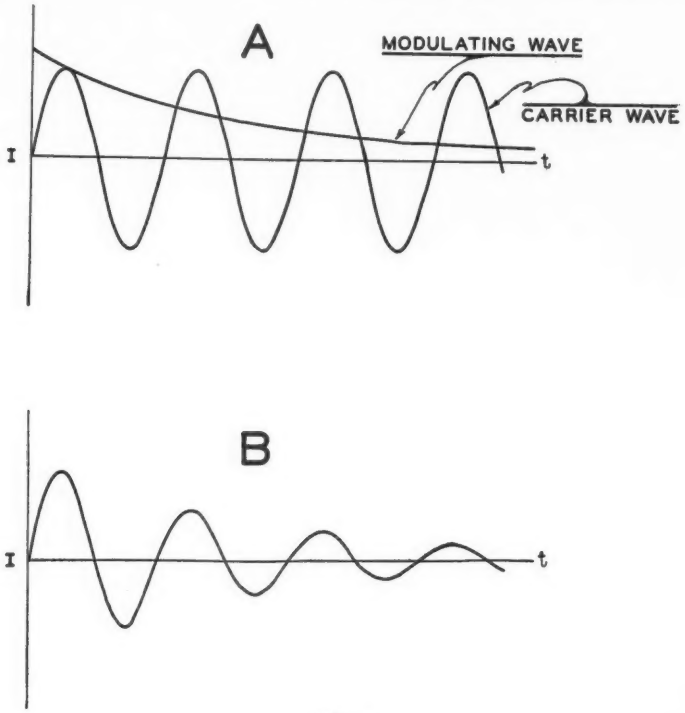

FIG. 1

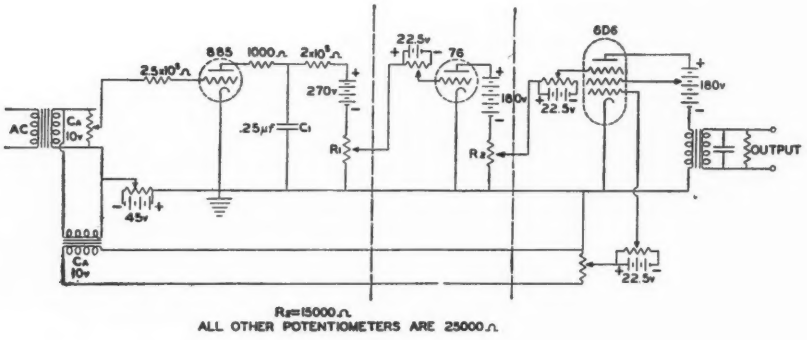

FIG. 2

testified to by anyone who has tried it. In certain tests, the mode of initiation of the transient is of great importance, and with the con-

ventional method this is difficult to control. Again, for the low-frequency waves such as are commonly met with in geophysical work, the values of L and C may easily be inconveniently large, and the switching arrangement conducive to rapid and easy alteration of the wave character become unreasonably complicated.

Recently,[1,2] other methods have been devised to produce suitable damped wave trains. Still another source of damped wave trains is here described, in the hope that, insofar as it may possess certain unique advantages, it may serve as a companion tool to the existing methods. It is believed that this circuit is more easily adapted to rapid variation of the wave character over a large range.

The general equation for a sinusoidal, damped wave train is

$$I = I_0 f(t) \sin \omega t,$$

where $f(t)$ is some function which approaches zero as t increases. Through long familiarity, we are accustomed to thinking of this function as an exponential one, and writing

$$I = I_0 e^{-\alpha t} \sin \omega t$$

for any damped wave train. The more general expression is given here, not academically, but because the circuit to be described will provide wave trains which have nearly any type of damping, as will be shown.

The point to be noted is that the equation shows the damped wave train to be the product of two functions of time, one of which is sinusoidal. Hence, if we can introduce into some mixing network two such time-variable components, their product should give us a damped wave train. The ordinary vacuum tube modulator circuit is such a network. Suppose we have as a carrier a sine wave, and modulate it with a wave such as is shown in Fig. 1a. We see that the modulated carrier (shown in Fig. 1b) is a typical damped wave train.

This is fundamentally the device used in the method under discussion. The carrier to be modulated is any sinuosidal wave of the desired frequency, such as that supplied by a beat frequency oscillator The modulating wave is obtained by utilizing a small thyratron of the type 885 in a typical sweep circuit arrangement. The complete circuit is shown in Fig. 2. It will be noted that the circuit diagram is separated by dashed lines into three sections.

[1] C. V. Rajam: *Journal of Scientific Instruments*, XIII, 331, October, 1936.

[2] Harold Washburn: Paper read at the Los Angeles meeting of the S.E.G.. March, 1937.

The first section is the thyratron sweep circuit. The operation of a thyratron for this purpose is quite well known and will be described but briefly. If we suppose that when $t=0$, the condenser C_1 is uncharged, the potential on the plate will be a function of time, such as that show in Fig. 3. The grid potential necessary for cutoff, i.e., so that no current will flow through the tube, will be roughly like that of Fig. 4a. Consider a fixed bias, E_{G0}, so that no current will flow through the tube. Then let us superpose an A.C. voltage on this bias

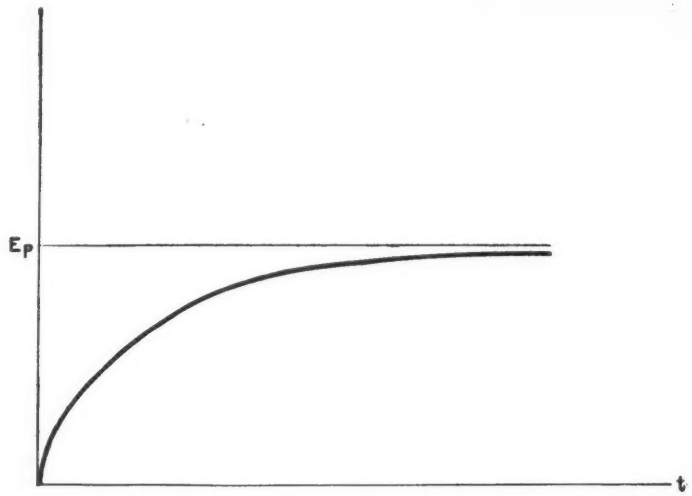

FIG. 3

such as in Fig. 4b. It can be seen that by varying the amplitude of this A.C. we can cause the tube to trip—that is, to pass current—after any given number of cycles. The thyratron tube has the unique property that, once a discharge is initiated, the current through the tube will flow as long as the plate potential is above a certain value. Hence, after the grid potential reaches a value so that the tube is tripped, the condenser V_1 is nearly totally discharged through the relatively low-resistance plate cathode circuit of the tube, and then charged up again. During the charging we have a P.D. across R_1 which, drawn as a function of time, will be roughly as shown in Fig. 5a.

This impulse is applied to the grid of the tube of the second section—the buffer, or inverter stage—which consists of a tube biased

so that it is well above cutoff: that is, so that when the above voltage is impressed on the grid we shall get a drop across the plate resistor R_2 like that of Fig. 5b.

This acts on the grid of the modulator tube in the third section, which is a 6-D-6 type pentode, biased so that when the modulator from the sweep circuit is absent, no current can flow through the tube. A.C. from the same source that supplied the thyratron is im-

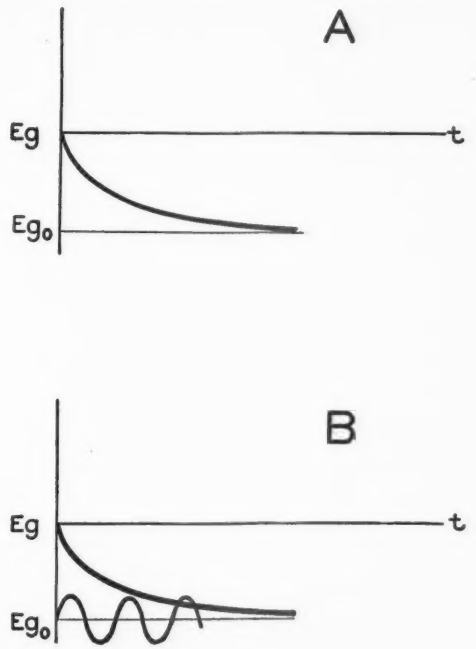

FIG. 4

pressed upon the control grid, while the modulating sweep voltage is impressed on the suppressor grid. The A.C. flows in the plate circuit only when the modulation is above a certain value, and the tube is usually operated in a region where the plate current grid voltage relation is linear. This linearity may be deviated from for certain purposes as will be mentioned later. The output is, then, a wave train which starts with a high amplitude and falls off according to the time slope of the modulating impulse. The capacity and resistance across the output transformer serve to load the plate circuit of the 6-D-6. The values are not critical—a capacity of 2 mfd. and a resistance of

15,000 ohms, in conjunction with a plate to 200 ohm transformer, were found to be satisfactory for the circuit now in operation. It is advantageous to use a variable resistance, as the load affects the characteristic operation of a 6-D-6, and by varying this load we can help to "balance" up the chosen wave train—i.e., attain a symmetry with respect to the time axis.

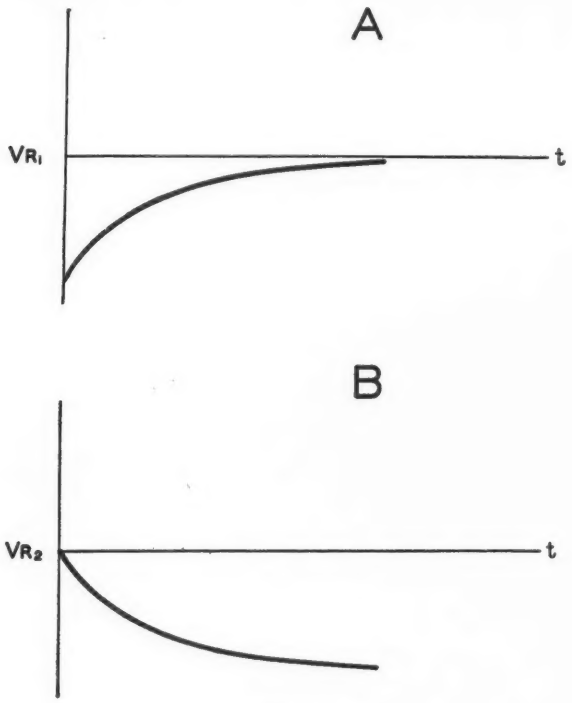

FIG. 5

No set procedure can be given for the operation of the circuit. It has been the writer's experience that one must more or less "play by ear" rather than "by note." In other words, after the circuit has been checked, connect the output to a cathode ray oscilloscope and vary the different controls and note the change in the wave pattern.

However, one or two points of possible help can be made. After the 885 is turned on, the D.C. grid bias should be varied so that with no superposed A.C. this bias is just sufficient to prevent the tube from tripping. This point is, of course, easily found by watching the tube for the characteristic glow that shows at each "trip." The A.C.

should then be adjusted so as to trip the tube at some fairly slow rate—say, five to ten times a second. These biases should then not be altered during the rest of the preliminary adjustment. The tap on R_1 should then be set at some value close to ground, and the grid bias on the 76 set at some point so that the modulating surge from the thyratron is passed through the tube. This can most readily be told by the use of a peak voltmeter across R_2, or by connecting the cathode ray oscilloscope here. The 6-D-6 biases are then adjusted until some pattern is obtained, and it will be found that the variation

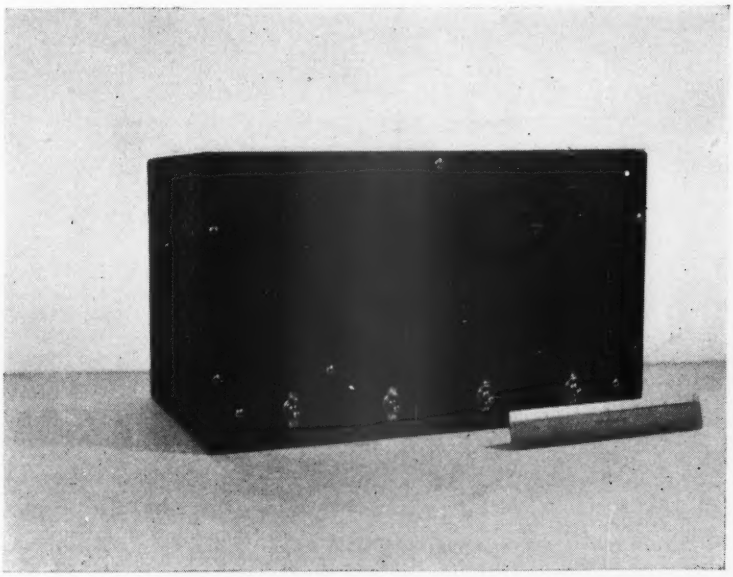

FIG. 6

of the character to nearly any desired form is a matter of practise of short duration. After the development of the present circuit, three such outfits have been built in this laboratory, and a half hour has sufficed in each case to attain the desired operation. Fig. 6 shows the completed unit in use at present.

It may be added that, in general, the potentiometers on the 76 control the damping; and those of the 6-D-6 and output, the character or type of damping. It has been found that once a setting is determined, the operation is very stable, and if the bias batteries are not allowed to deteriorate by being left connected to the potentiom-

eters, the circuit may be put into operation after being idle several weeks, and the wave train will be the same as that previously obtained to an excellent approximation, at least.

Fig. 7 shows the two components of the wave train, the carrier and the modulating wave. It was made by taking a double exposure of the two types of waves as seen on a cathode ray screen. For the photograph of the modulating wave, the carrier was entirely suppressed by controlling the carrier potentiometer; for a picture of the carrier wave, the modulating voltage was cut off.

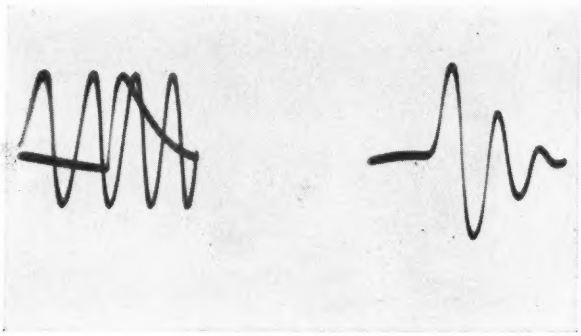

FIG. 7

Figs. 8 and 9 illustrate some of the various types of damped wave trains that have been obtained with this circuit. It can be seen that in some of the cases the damping is by no means exponential or logarithmic. It is possible, in fact, to get transients whose damping is anything from straight line "saw-tooth" to parabolic.

Fig. 10 shows how this apparatus is used in the analysis of experimental filter circuits. It was desired to make some rather rapid tests of these circuits in order to determine the relative values of the phase shift and to find out whether or not this phase shift was large enough to cause a measurable time delay which would be a function of frequency; and what effect, if any, the shape of the start of the wave had on the initial phase shift. The method used was to take a picture of the unfiltered damped wave train on the cathode ray screen; then, leaving the camera in place, to pass this wave train through the filter and make a second exposure on the same film. The relative positions of the filtered and unfiltered waves will be a measure of the phase difference of the two waves. This difference can be easily measured

by means of a protractor. The only justification for doing this in such a fashion is that it was tried with known filter sections, using

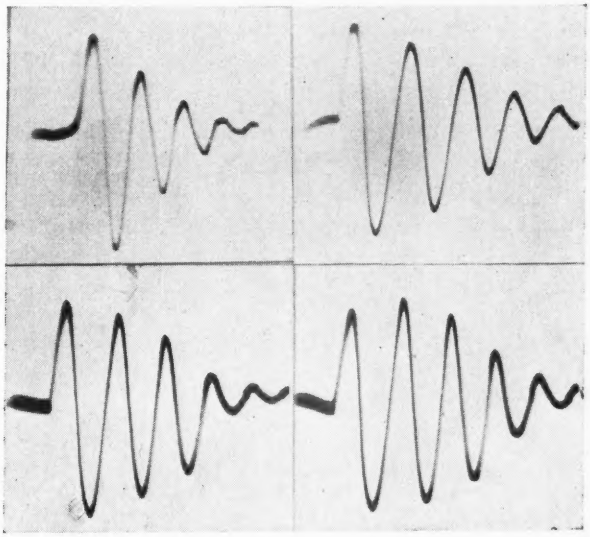

FIG. 8

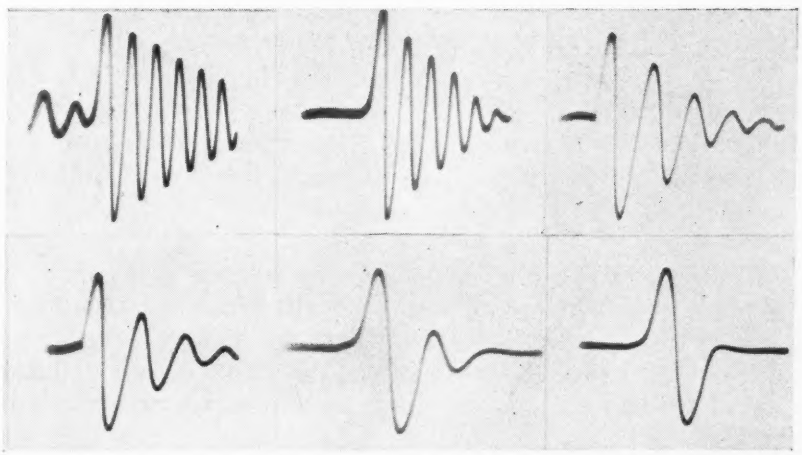

FIG. 9

steady state current, and the measured phase difference was found to be in very close agreement with that computed. Furthermore, our experience has shown that the image will remain fixed on the cathode

ray screen without shifting for a period of several hours; and also repeated photographs of the filtered and unfiltered wave trains have been made and the phase difference checked. Agreement within the error of measurement was found in every case.

Fig. 10a shows the unfiltered wave. Fig. 10b shows the filtered wave after having passed through a high-pass filter, which was coupled by means of a tube—hence the inversion. Fig. 10c shows the same wave after having been passed through a low-pass filter. In Fig. 10d the same wave is shown after having been passed through the two filters connected in series, so that they formed an effective band-pass

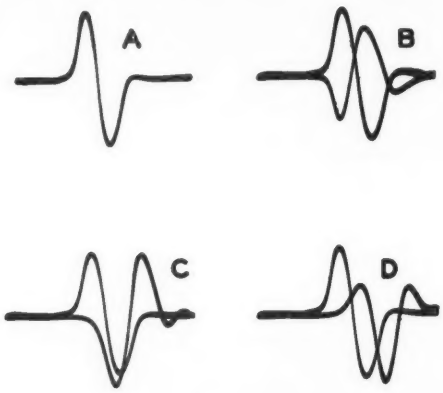

FIG. 10

filter system. There are more accurate and rigorous methods of determining phase shift; but as a rough experimental check, this method is indeed a labor-saving device, especially when it is desired to use various highly damped transients.

Another use to which this circuit has been applied is in the making of synthetic films for test purposes. The output from the damped wave train can easily be amplified, and this amplified output may be applied to any recording system desired. We have found that the analyzed synthetic records made in such fashion will reproduce quite accurately the imposed wave train.

The writer is indeed glad of this opportunity to acknowledge his indebtedness to C. H. Johnson, who suggested the basic method, and to B. F. McNamee for his valued advice and assistance in the development of the circuit.

A NOTE ON THE PROPAGATION OF SEISMIC WAVES*

MORRIS MUSKAT†

ABSTRACT

It is suggested that in the computation of theoretical time-distance curves for seismic wave propagation a more tractable form of analysis is obtained if the depth is expressed as a power series in the velocity than when the converse but more conventional method is used. Several illustrations of this procedure are given.

It is well known that the trajectory of a wave in an elastic medium whose velocity varies only in one direction (y) may be expressed parametrically in the form:

$$x = \int_{y_0}^{y} [v/(\alpha^2 - v^2)^{1/2}]dy; \; t = \alpha \int_{y_0}^{y} [v(\alpha^2 - v^2)^{1/2}]^{-1}dy, \quad (1)$$

$$\alpha = v/\sin\theta = v_0/\sin\theta_0,$$

where v, θ are the velocity and angle with the vertical at the depth y, v_0, θ_0 being those at $y=y_0$.

For a complete seismic path beginning at the origin ($x_0=y_0=0$) and ending at the time $\bar{t}$ at $x=\bar{x}$, $y=0$, we have:

$$\bar{x} = 2\int_{0}^{y_{max}} [v/(\alpha^2 - v^2)^{1/2}]dy; \; \bar{t} = 2\alpha \int_{0}^{y_{max}} [v(\alpha^2 - v^2)^{1/2}]^{-1}dy, \quad (2)$$

where:

$$v(y_{max}) = \alpha, \quad (2a)$$

provided that the plane of y_{max} is not a plane of discontinuity in $v(y)$.

As:

$$\frac{d\bar{t}}{d\bar{x}} = \frac{1}{\alpha}, \quad (3)$$

a time-distance curve, $\bar{x}=\bar{x}(\bar{t})$, permits an evaluation (analytical or graphical) of $\bar{t}$ or $\bar{x}$ as a function of α and hence an inversion of the Eqs. (2) by the Abel inversion formula so as to give the $v(y)$ corresponding to the observed time-distance curve. Although this procedure affords a closed and complete solution to the analytical problem of interpreting time-distance curves in terms of the velocity distribu-

* Paper read by title, Fall Meeting, Houston, Nov. 19, 1937.

† Gulf Research & Development Company, Pittsburgh, Penna. Introduced by E. A. Eckhardt.

tions in the elastic media traversed by the seismic waves, it is nevertheless of value, as in other problems of geophysical interpretation, to work out the results to be expected on a number of ideal hypotheses of velocity distribution so as to provide at least a qualitative means of judgment of the significance of a given time-distance curve by comparison with previously computed "typical" curves.

In deriving such "typical" curves one simply needs to assume the form of $v(y)$ and evaluate the integrals of Eq. (1). However, the immediate substitution in Eqs. (2) of even simple functions for $v(y)$ leads to quite difficult and almost intractable integrals. Thus the quadratic expression in y gives elliptic integrals, and cubic expressions for $v(y)$ appear almost hopelessly difficult to treat rigorously.

Although this analytical difficulty is quite real, useful results may nevertheless be obtained by a slight change in the method. For if the above procedure be reversed and y expressed explicitly in terms of v, as is formally done in developing the integral equation solution, the problem becomes considerably simplified.

Thus with the notation:

$$y = y(v); \quad \Phi(v) = \frac{dy}{dv}, \tag{4}$$

Eq. (2) becomes:

$$\bar{x} = 2\int_{v_0}^{\alpha} [v\Phi(v)/(\alpha^2 - v^2)^{1/2}]dv; \quad \bar{t} = 2\alpha\int_{v_0}^{\alpha} [\Phi(v)/v(\alpha^2 - v^2)^{1/2}]dv. \tag{5}$$

Of course, the mere notation of Eq. (4) cannot enable the evaluation of an integral otherwise intractable, and for cases where the direct use of Eqs. (2) is difficult so will be the use of Eqs. (5). Nevertheless, the Eqs. (4) and (5) suggest the use of and are particularly convenient for the treatment of velocity variations that are of more physical interest than those naturally suggested by the form of the Eqs. (2). Thus we not only find that whereas the general polynominal function $v(y) = \sum_0^m a_n y^n$ cannot be treated by Eqs. (2), the function $y = \sum_0^m a_n v^n$ is readily discussed by means of Eqs. (5). Moreover, from a physical point of view the former expression implies for great depths an increasingly rapid growth of the velocity with depth (assuming $a_m > 0$) whereas the latter corresponds to the more reasonable picture of a decreasing rate of growth of the velocity at great depths.

Considering first, then, the general case where the velocity polynomial is an infinite series and has the value v_0 at the surface, $y=0$, we may write:

$$y = (v - v_0)\sum_0^\infty a_n v^n;\quad \Phi(v) = \sum_0^\infty (n+1)\{a_n - v_0 a_{n+1}\}v^n. \tag{6}$$

Putting this in Eqs. (5), it is readily found that:

$$\bar{x} = 2\sum_0^\infty (n+1)(a_n - v_0 a_{n+1})\alpha^{n+1} I_{n+1}(\theta_0), \tag{7}$$

$$\bar{t} = 2\sum_0^\infty (n+1)(a_n - v_0 a_{n+1})\alpha^{n} I_{n-1}(\theta_0), \tag{8}$$

where:

$$I_n = \frac{1\cdot 3\cdots(n-3)(n-1)}{2\cdot 4\cdots(n-2)n}(\pi/2 - \theta_0) + \frac{\sin 2\theta_0}{2n}\Big[\sin^{n-2}\theta_0 + \frac{n-1}{n-2}\sin^{n-4}\theta_0 + \frac{(n-1)(n-3)}{(n-2)(n-4)}\sin^{n-6}\theta_0 + \cdots + \frac{(n-1)(n-3)\cdots 3}{(n-2)(n-4)\cdots 2}\Big] : n \text{ even}, \tag{9}$$

$$I_n = \frac{\cos\theta_0}{n}\Big[\sin^{n-1}\theta_0 + \frac{n-1}{n-2}\sin^{n-3}\theta_0 + \cdots + \frac{(n-1)(n-3)\cdots 2}{(n-2)(n-4)\cdots 3}\Big] : n \text{ odd}, \tag{10}$$

$$I_0 = (\pi/2 - \theta_0);\quad I_{-1} = \log \operatorname{ctn}(\theta_0/2). \tag{11}$$

The nature of the functions $I_n(\theta_0)$ may be seen from Fig. 1, where they are plotted for $n=0$ to 5.

Eqs. (7)–(11) provide the parametric solution for the time-distance curve for any velocity distribution of the form given by Eq. (6).[1] If the trajectories or wave fronts are desired one need only

[1] This same method and analogous formulas can of course be used in the case of a spherical earth where the velocity varies only with the radial distance. Moreover, it can be readily extended to the case of discontinuous velocity variations, where different expansions, as of Eq. (6), are preassigned for each stratum, provided the value of α does not lead to total reflection at one of the planes of discontinuity.

change the upper limits of the integrals in Eqs. (5) to $v(y)$ and remove the factors 2. The curves for $\bar{x}$ or $\bar{t}$ vs. the maximum depth $y_{\max}$ may also be obtained by computing first from Eqs. (7) and (8) $\bar{x}$ and $\bar{t}$ as a function of α (or θ_0) and then calculating the corresponding $y_{\max}$ by means of Eq. (6). Finally it may be noted that the above procedure may also be applied to reflection shooting time-distance curves, although the increased number of parameters will require considerably more extended calculations than Eqs. (7) and (8). For the simplest case, however, where the elastic waves are reflected at the first interface, with a velocity distribution in the upper layer given by Eq. (6), one may still formally use Eqs. (7) and (8) but with the $I_n(\theta_0)$ replaced by $I_n(\theta_0)-I_n(\theta_h)$ where θ_h is the angle of reflection and is given by $\theta_h=\sin^{-1} v_h/\alpha$, and $v_h=v(y=h)$ by Eq. (6).

As examples of the application of Eqs. (7) and (8) we first choose the case of a quadratic variation of depth with velocity, i.e.:

$$(1): y = (v - v_0)(a_0 + a_1 v). \tag{12}$$

Then by Eqs. (7) and (8) we have:

$$\bar{x} = 2(a_0 - v_0 a_1)\alpha I_1 + 4a_1\alpha^2 I_2$$

$$= 2a_0 v_0 \operatorname{ctn}\theta_0 + \frac{2a_1 v_0^2}{\sin^2\theta_0}\left(\frac{\pi}{2} - \theta_0\right), \tag{13}$$

$$\bar{t} = 2(a_0 - v_0 a_1) I_{-1} + 4a_1\alpha I_0$$

$$= 2(a_0 - v_0 a_1)\log \operatorname{ctn}\frac{\theta_0}{2} + \frac{4a_1 v_0}{\sin\theta_0}\left(\frac{\pi}{2} - \theta_0\right). \tag{14}$$

Because of the transcendental dependence of $\bar{x}$ and $\bar{t}$ on θ_0 or α one cannot eliminate them and obtain $\bar{x}(\bar{t})$ explicitly.[2] The numerical work of finding $\bar{x}(\bar{t})$ may, nevertheless, be carried through quite readily.

Another case of more physical interest in that it permits the velocity to grow only asymptotically to a limiting value v_1, and corresponding to a choice of the a_n in Eq. (6) so as to be summable in closed form, is given by:

$$(2): y = a \log\frac{(v_1 + v)(v_1 - v_0)}{(v_1 - v)(v_1 + v_0)}; \Phi(v) = \frac{2av_1}{v_1^2 - v^2}, \tag{15}$$

[2] In the case of a linear variation of v with $y(a_1=0)$ one obtains immediately: $\bar{x}=2a_0v_0\sinh\bar{t}/2a_0$; $y_{\max}=\sqrt{a_0^2v_0^2+\bar{x}^2}/4-a_0v_0$. Likewise for the reflection problem for this case the procedure indicated above gives the result, also well known: $\cosh\bar{t}/2a_0 = \{\bar{x}^2+4a_0^2(v_h^2+v_0^2)\}/2a_0^2v_0v_h$, where $v_h=v_0+h/a_0$.

so that:

$$\bar{x} = 4av_1 \int_{v_0}^{\alpha} \frac{vdv}{(v_1^2 - v^2)\sqrt{\alpha^2 - v^2}} = \frac{4a\beta \sin \theta_0}{\sqrt{\beta^2 \sin^2 \theta_0 - 1}} \tan^{-1} \frac{\cos \theta_0}{\sqrt{\beta^2 \sin^2 \theta_0 - 1}}, \tag{16}$$

$$\bar{t} = 4a\alpha v_1 \int_{v_0}^{\alpha} \frac{dv}{v(v_1^2 - v^2)\sqrt{\alpha^2 - v^2}} = \frac{\bar{x}\alpha}{v_1^2} + \frac{4a}{v_1} \tanh^{-1} \cos \theta_0, \tag{17}$$

β representing v_1/v_0.

As a final example we shall take another case in which the velocity approaches asymptotically a limiting value as the depth y increases indefinitely, but one in which the approach is hyperbolic rather than exponential, as in the last illustration. Thus we set:

$$(3): \qquad y = a\left(\frac{v - v_0}{v_1 - v}\right); \qquad \Phi(v) = \frac{a(v_1 - v_0)}{(v_1 - v)^2}. \tag{18}$$

By Eqs. (5) we have:

$$\left.\begin{aligned} \bar{x} &= 2a(v_1 - v_0) \int_{v_0}^{\alpha} \frac{vdv}{(v_1 - v)^2\sqrt{\alpha^2 - v^2}} = \frac{2a(\beta - 1) \sin \theta_0}{\beta^2 \sin^2 \theta_0 - 1} \\ &\cdot \left[\frac{\beta \cos \theta_0}{\beta - 1} + \frac{1}{\sqrt{\beta^2 \sin^2 \theta_0 - 1}}\right. \\ &\cdot \left.\left\{\frac{\pi}{2} - \sin^{-1} \frac{\beta \sin^2 \theta_0 - 1}{(\beta - 1) \sin \theta_0}\right\}\right], \end{aligned}\right\} \tag{19}$$

$$\left.\begin{aligned} \bar{t} &= 2a\alpha(v_1 - v_0) \int_{v_0}^{\alpha} \frac{dv}{v(v_1 - v)^2\sqrt{\alpha^2 - v^2}} = \frac{2a(\beta - 1)}{v_0\beta^2(\beta^2 \sin^2 \theta_0 - 1)} \\ &\cdot \left[\frac{\beta \cos \theta_0}{\beta - 1} + \frac{(2\beta^2 \sin^2 \theta_0 - 1)}{\sqrt{\beta^2 \sin^2 \theta_0 - 1}}\right. \\ &\cdot \left.\left\{\frac{\pi}{2} - \sin^{-1} \frac{\beta \sin^2 \theta_0 - 1}{(\beta - 1) \sin \theta_0}\right\}\right] \\ &+ \frac{2a(\beta - 1)}{v_0\beta^2} \log \operatorname{ctn} \theta_0/2, \end{aligned}\right\} \tag{20}$$

where again $\beta = v_1/v_0$.

It should again be emphasized that in such cases as (2) and (3) where we would have:

$v = v_1 \tanh (y/2a + \tanh^{-1} v_0/v_1)$; and $v = v_1 - a(v_1 - v_0)/(y + a)$,

the possibility of integrating Eq. (5) necessarily implies the integrability also of Eq. (2). However, the form of Eqs. (5) does give simpler integrals, especially in such cases as (2) where $\Phi(v)$ is an even function of v.

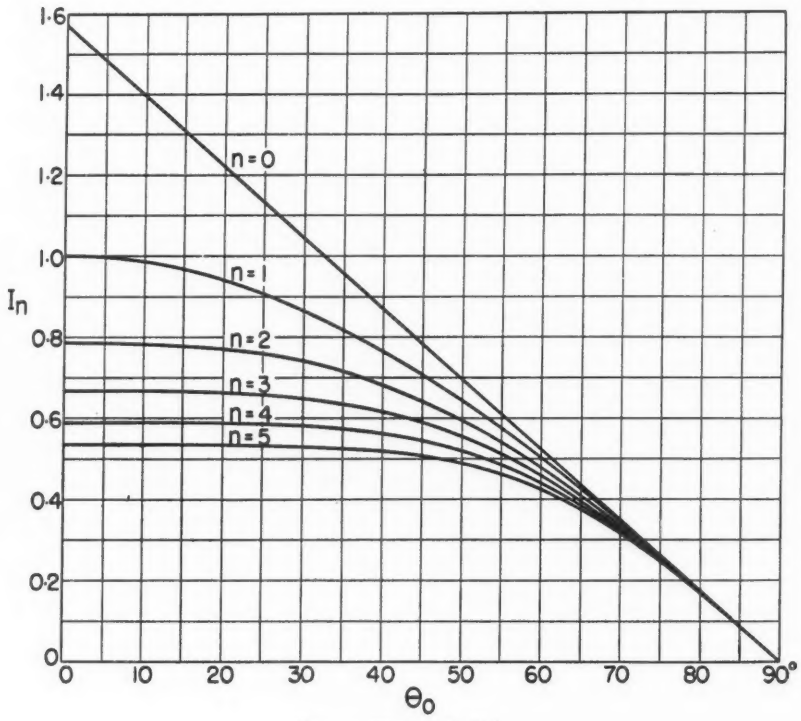

FIG. 1. Plot of $I_n(\theta_0)$.

For purposes of numerical illustration we shall choose the following constants:

$$\left.\begin{aligned}
&(1)\ (\text{Eq. } (12)):\ a_0 = 2;\ a_1 = 0;\ v_0 = 5 \times 10^3 \text{ ft./sec.}\\
&(2)\ (\text{Eq. } (12)):\ a_0 = 2;\ a_1 = 2 \times 10^{-4};\ v_0 = 5 \times 10^3 \text{ ft./sec.}\\
&(3)\ (\text{Eq. } (15)):\ a = 10^4;\ v_0 = 5 \times 10^3 \text{ ft./sec.};\ v_1 = 4v_0\\
&(4)\ (\text{Eq. } (18)):\ a = 10^4;\ v_0 = 5 \times 10^3 \text{ ft./sec.};\ v_1 = 4v_0.
\end{aligned}\right\} \quad (21)$$

The velocity-depth curves for these cases are plotted in Fig. 2.

The corresponding time-distance curves are given in Fig. 3. The maximal penetrations are plotted versus the ranges of the seismic paths in Fig. 4, and the angles of incidence or emergence in Fig. 5.

The general features of these curves are to be expected from the nature of the assumed velocity distributions and need no detailed

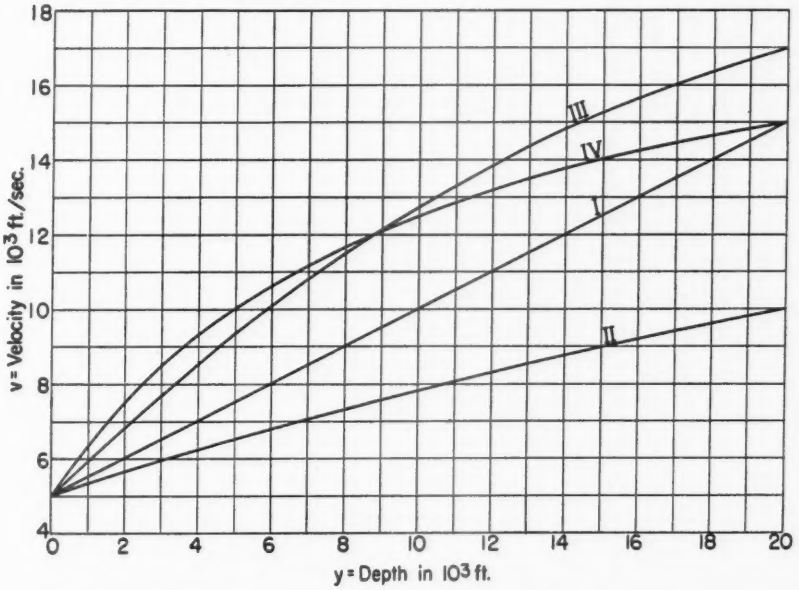

FIG. 2. Velocity-depth distributions of Eq. (21). I: Linear increase of depth with velocity; II: Quadratic increase of depth with velocity; III: Logarithmic increase of depth with-velocity; IV: Hyperbolic increase of depth with velocity.

discussion. Thus, it is clear from Fig. 3 that the times of arrival for a given range decreases as the average velocity of the media traversed increases, the resulting time-distance curves showing otherwise no features particularly characteristic of the analytic form of the velocity -depth distribution.

As is to be expected from general considerations, Fig. 5 shows that for a given angle of emergence the range is larger for smaller rates of velocity growth. Furthermore it is seen from curves III and IV that the angle approaches asymptotically a limiting value as the range becomes infinite for the cases where the velocity does not grow indefinitely with the depth. The general decrease of the angle of

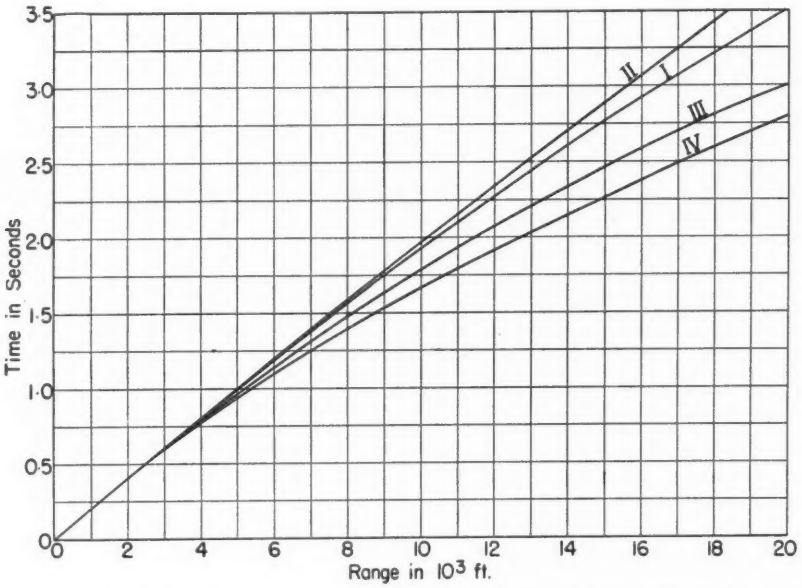

FIG. 3. Time-distance (range) curves for the velocity-depth distributions of Fig. 2.

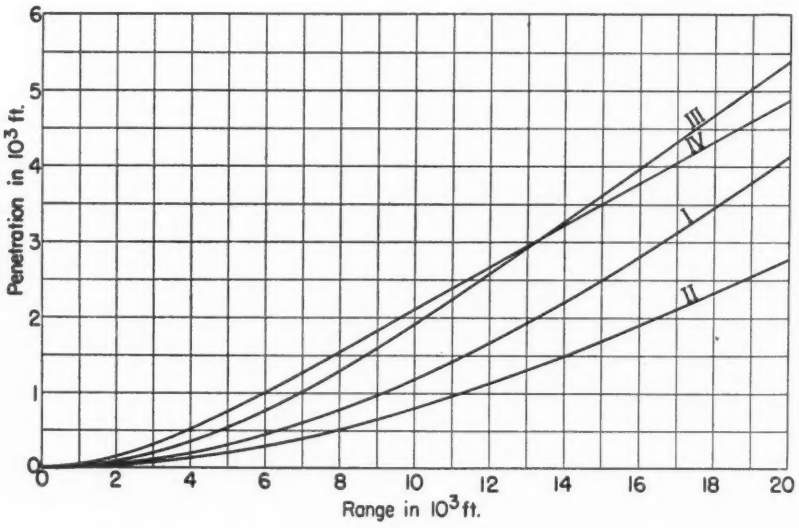

FIG. 4. Maximum penetration versus range for the velocity-depth distributions of Fig. 2.

emergence with increase in range is evidently directly related to the fact that the time-distance curves shown in Fig. 3 are convex upwards,[3] and may be contrasted with the range vs. angle variation given by $\bar{x}=\pi \sin\theta_0$, corresponding to the time-distance curve: $\bar{t}=\pi/2(x^2/\pi^2+1)$ which follows from the velocity function $v^2=1/(1-y^2)$. This latter

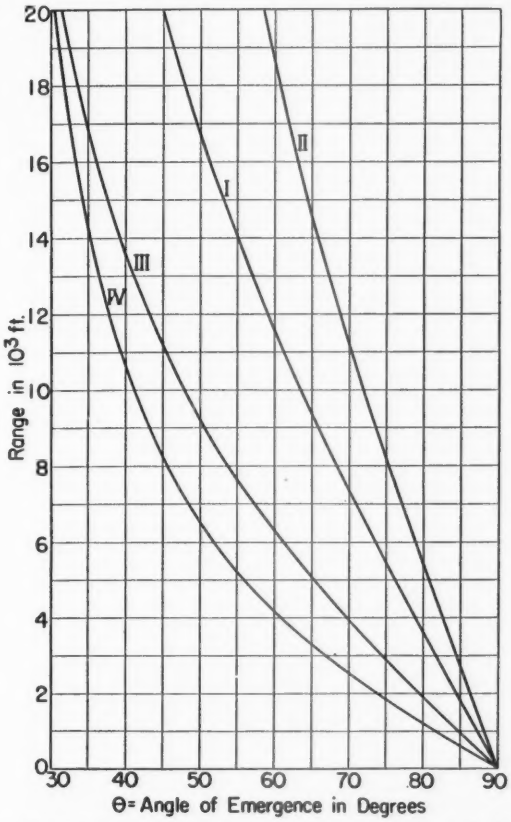

FIG. 5. Range versus angle of emergence of seismic paths in media with velocity-depth distributions of Fig. 2.

gives, of course, a second arrival "reversed segment,"[4] although for small depths the velocity variation differs but slightly from that given by: $v^2=1+y^2$, which leads to a normal first arrival time-distance curve defined by: $\bar{x}=2\alpha E$; $\bar{t}=2K$, where K and E are the complete

[3] From Eq. (3) it immediately follows that: $d\theta_0/d\bar{x}=v_0 \sec\theta_0\, d^2\bar{t}/d\bar{x}^2$.

[4] Cf. L. B. Slichter, Physics 3, 273, 1932.

elliptic integrals of the first and second kind with modulus equal to $\cos \theta_0$. The fact that in this last case there is a dead zone for the refracted paths for ranges less than π is clearly due to the infinitely slow velocity growth at the surface, and the associated singularity of the integrands of Eq. (5) for $\alpha = v_0 = 1$.

The writer is indebted to Mr. L. W. Gardner and Mr. W. C. Lamb of this Laboratory for helpful discussions and to Dr. Paul D. Foote and Dr. E. A. Eckhardt for permission to publish this note.

A SET OF CURVES TO ASSIST IN THE INTERPRETATION OF THE THREE LAYER RESISTIVITY PROBLEM*

W. W. WETZEL† AND HOWARD V. McMURRY†

ABSTRACT

A set of apparent resistivity curves for the Wenner electrode configuration is given. This set covers a wide range of variations of layer thickness and resistivity for the three-layer earth. A method for interpretation of field data is outlined.

One of the common problems encountered in applications of electric methods to geophysical prospecting is the interpretation of resistivity data associated with the so-called three-layer earth. Even in such simple situations as the determination of bed rock depth where the geological system is composed of two layers the electric system, because of ground moisture, will frequently exhibit three layers. Unfortunately the equations which give the potential distribution about an electrode on the multiple-layer earth either involve infinite series with undesirable convergence properties or contain integrals which cannot be evaluated in simple analytical form. While the two-layer problem has been solved completely,[1] only sporadic examples of theoretical three-layer resistivity curves have been published and there is no comprehensive set of data available for guidance in interpretation of the three-layer case. Possibly due to this deficiency there has been a tendency on the part of some to use a double application of the simpler two-layer results in the interpretation of three-layer data. This process as it will appear is not generally permissible.

While it has been known for some time that the best possible method of interpretation required a set of standard curves to which the field curves could be matched, the labor involved in determining such a set of standards has, in one way or another, prevented the publication of the necessary data. L. B. Slichter pointed out some time ago that the application of the Differential Analyzer to the calculation would reduce the time element considerably and it was at his suggestion the present work was undertaken using that machine.

As apparent resistivity is an artificial concept depending upon the

* Received, May, 1937.

† Department of Geology, Massachusetts Institute of Technology, Cambridge, Mass. Introduced by M. M. Slotnick.

[1] Irwin Roman, Some Interpretations of Earth-resistivity Data. A.I.M.E. Trans., *110*, 183 (1934).

electrode configuration it was thought best to solve for the potential which could then be used to determine the resistivity for any electrode arrangement. For the purposes of this paper the Wenner configuration consisting of four electrodes on the same straight line and equally spaced was chosen because of its general use.

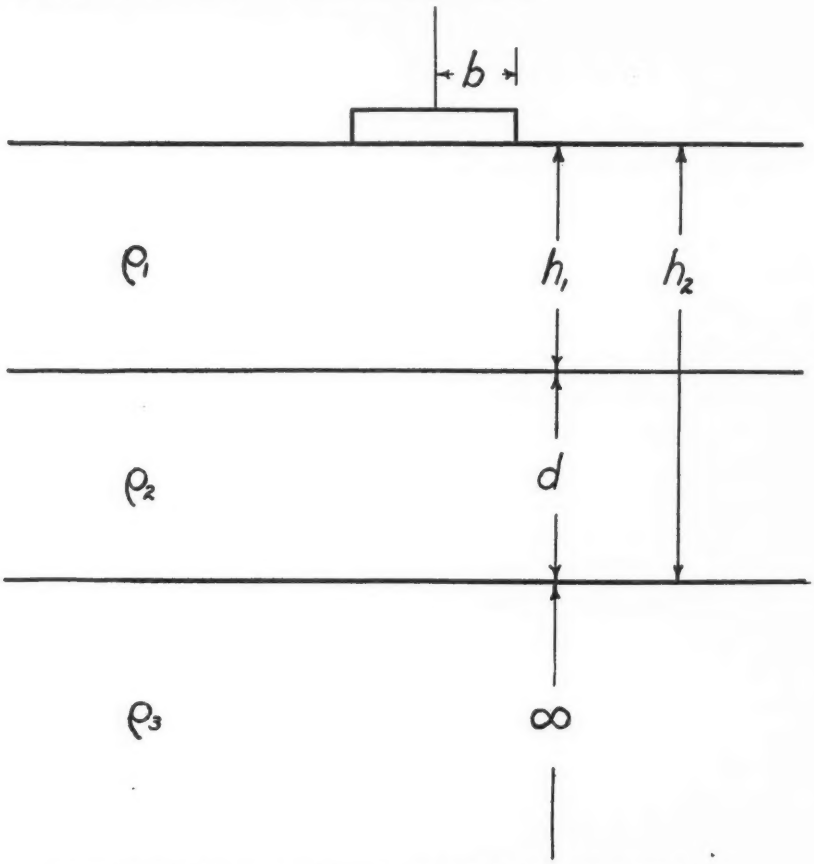

FIG. 1. Showing the layer thicknesses and resistivities for the three-layer case.

The expression[2] for the potential ϕ at a distance r from a disk electrode of radius b on the surface of an earth of any number of layers is

$$\phi(r) = \frac{C\rho_1}{2\pi}\left[\frac{1}{r} + \int_0^\infty \frac{\sin \lambda b}{\lambda b} J_0(\lambda r) 2B_1(\lambda) d\lambda\right], \qquad (1)$$

[2] L. B. Slichter, The Interpretation of the Resistivity Prospecting Method for Horizontal Structures, Physics, *4*, 307 (1931).

where ρ_1 is the specific resistivity of the upper layer, C the current, $J_0(\lambda r)$ is the Bessel function of zero order and B_1 is a function of the constants of the multiple-layer earth under the electrode. It is assumed that the layers are homogeneous and of uniform thickness. For the case of a three-layer earth illustrated in Fig. 1 this function B_1 is given by

$$B_1(\lambda) = \frac{k_1 e^{-2\lambda h_1} + k_2 e^{-2\lambda h_2}}{1 - k_1 e^{-2\lambda h_1} - k_2 e^{-2\lambda h_2} + k_1 k_2 e^{-2\lambda d}},$$

where $$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad \text{and} \quad k_2 = \frac{\rho_3 - \rho_2}{\rho_3 + \rho_2}.$$

The relationship between the apparent resistivity ρ_a and the potential ϕ is given by the following expression

$$\rho_a = \frac{2\pi a}{C}\left[2\phi(a) - 2\phi(2a)\right]. \qquad (2)$$

Here a is the common distance between the four electrodes.

The evaluation of Equation (1) was done on the Differential Analyzer.[3] The ϕ values were then plotted to detect accidental errors. Values of ϕ taken from these plots were substituted in Equation (2) to determine ρ_a. Curves of the apparent resistivity were plotted in several kinds of families and any obvious irregularities investigated. Whenever possible the asymptotic two-layer curves were used as guides in detecting systematic errors.[4] Frequently these guide curves were useless in the region of the calculation and only by extension of the work to very much larger values of a would the asymptotic curves be approached.

It is impossible to assign a probable error to the calculated values. Of the three-layer ratios used the greatest numerical errors should occur in the values for $h_1 : d = 1 : 3$. To check the accuracy of our work in this worst possible case the series expression for the apparent resistivity was evaluated for eight points. These points have been plotted as crosses on Figs. 2 and 4. The numerical values for the two sets of calculations are given in Table 1. Since depth determinations by the resistivity method can at best be made to only about 10 per cent accuracy these numerical differences are considered unimportant. It would be a senseless expenditure of effort to have tripled the time

[3] V. Bush J. Frank. Inst., *212*, 447 (1931).

[4] J. N. Hummel, Zeitsch. f. Geophysik, *5*, 89 (1929).

of an already lengthy calculation to obtain standard values 1 per cent accurate to be used in applications which for other reasons are limited to 10 per cent accuracy.

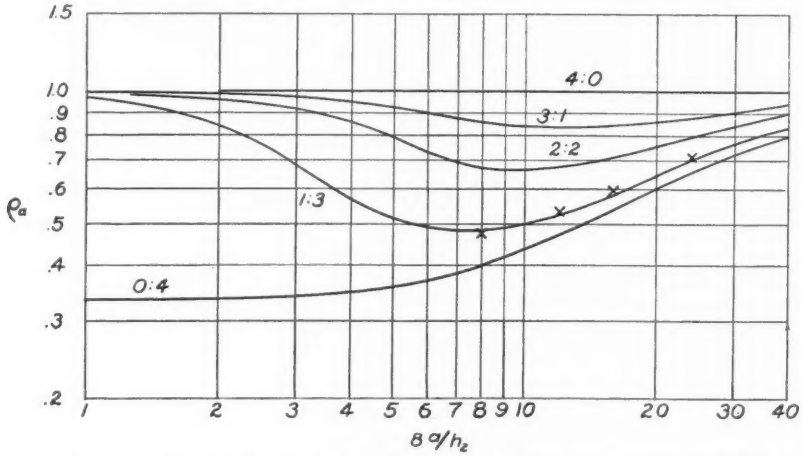

FIG. 2. Showing the effects on the apparent resistivity of varying the values of h_1. $\rho_1:\rho_2:\rho_3$ as $1:\frac{1}{3}:1$ for this family. The labels on the curves give the $h_1:d$ distances.

Tables 2, 3 and 4 list the calculated values of ρ_a as a function of the several varying factors. Although the last figure in the tabulated

TABLE I

$8\,a/h_2$	*Series Solution*	*Integral Solution*	
8	1.63	1.59	$\rho_1:\rho_2:\rho_3=1:3:1/100$
12	1.29	1.28	
16	.89	.87	$h_1:d=1:3$
24	.37	.36	
8	.476	.484	$\rho_1:\rho_2:\rho_3=1:1/3:1$
12	.535	.521	
16	.596	.579	
24	.714	.695	$h_1:d=1:3$

data may have no significance it is given to assist in drawing smooth curves. Figures 2, 3 and 4 show three families of curves illustrating the effect on the resistivity function of varying certain constants one at a time.

Several very general comments on interpretation methods which have been proposed from time to time may be made at this point. a) All three families illustrated show a change of position of the maxima or minima with any variation of conditions, h_2 remaining constant. It is obvious that there is no one-to-one correlation of maxi-

mum, minimum or inflection point with the depth to the second interface. b) The effect of a vertical discontinuity of specific resistivity is to produce a broad fold in the curve and not sharp narrow folds. This

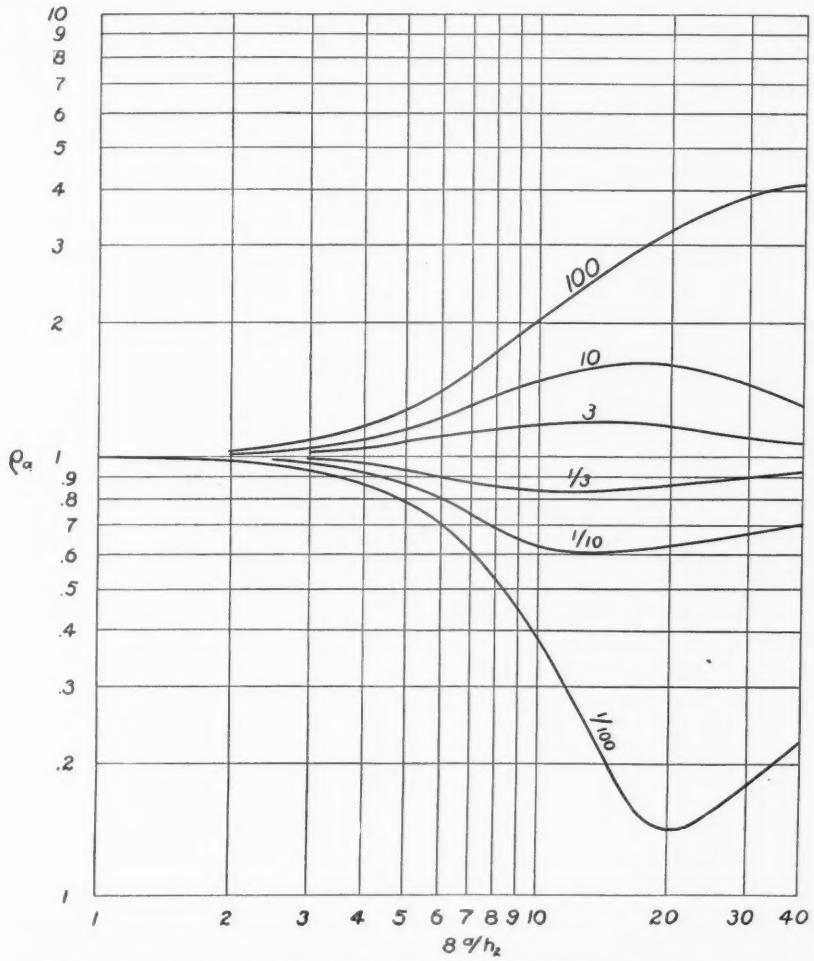

FIG. 3. Illustrates the effect on the apparent resistivity of varying ρ_2 all other factors remaining constant. $h_1:d$ as 3:1 and $\rho_1=\rho_3=1$. The labels on the curves give the values of ρ_3.

fact is of course independent of the scale of the problem. A vertical discontinuity at a depth of a thousand feet would produce a fold about a thousand feet, not several tens of feet, broad. Such small peaks are

the result of shallow horizontal discontinuities and cannot be associated with deep structure. c) A superficial resemblance of many three-layer curves to two-layer curves may be noticed. The difference is apparent only when the suspected curve is fitted to a nest of two-layer curves, when it will be found that they cannot be made to match over any considerable distance. However, fair approximations to the

TABLE II

LISTING VALUES OF ρ_a FOR $h_1=6$, $h_2=8$

$\rho_1:\rho_2:\rho_3$ \ $8a/h_2$	2	4	6	8	10	15	20	25	30	40
1:1/100:1/10	.979	.869	.693	.518	.366	.136	.0785	.0763	.0810	.0861
1/3	.979	.869	.698	.527	.376	.160	.1075	.1150	.1275	.1490
1	.979	.870	.706	.536	.388	.190	.144	.157	.178	.227
3	.979	.871	.708	.542	.393	.194	.158	.177	.205	.266
10	.979	.871	.709	.544	.396	.203	.166	.188	.218	.285
1:1/10:1/100	.970	.848	.674	.510	.380	.148	.062	.032	.0196	.0116
1/3	.983	.898	.761	.634	.529	.308	.356	.345	.341	.340
1	.986	.918	.808	.691	.621	.605	.625	.645	.665	.700
3	.987	.927	.831	.750	.719	.745	.865	1.00	1.13	1.350
10	.988	.933	.845	.773	.745	.835	1.025	1.23	1.425	1.820
100	.989	.936	.854	.789	.763	.880	1.11	1.36	1.620	2.150
1:1/3:1/100	.970	.865	.713	.550	.416	.182	.077	.035	.0196	.013
1/10	.980	.894	.765	.623	.495	.278	.186	.147	.129	.112
1	.989	.949	.898	.855	.832	.830	.850	.873	.895	.930
3	.997	.994	.998	1.030	1.080	1.26	1.44	1.60	1.74	1.98
10	.999	1.000	1.020	1.092	1.223	1.61	2.00	2.38	2.71	3.28
100	1.000	1.023	1.060	1.148	1.284	1.76	2.28	2.86	3.42	4.52
1:3:1/100	.990	.980	.970	.900	.815	.550	.323	.182	.106	—
1/10	1.000	.995	.980	.945	.875	.650	.430	.294	.212	.128
1/3	1.000	1.020	1.025	1.005	.965	.840	.680	.540	.460	.382
1	1.010	1.045	1.100	1.150	1.180	1.20	1.175	1.13	1.10	1.08
10	1.025	1.135	1.345	1.590	1.850	2.49	3.05	3.55	4.02	4.82
100	1.04	1.170	1.42	1.73	2.09	3.01	3.99	4.99	5.98	7.95
1:10:1/100	1.01	1.05	1.14	1.250	1.280	1.21	1.01	.775	.558	.230
1/10	1.01	1.06	1.16	1.265	1.300	1.26	1.07	.850	.650	.350
1/3	1.02	1.08	1.18	1.300	1.365	1.36	1.21	1.04	.895	.653
1	1.02	1.11	1.24	1.375	1.480	1.625	1.610	1.54	1.45	1.30
3	1.04	1.150	1.30	1.495	1.675	2.05	2.32	2.53	2.67	2.86
100	1.04	1.19	1.49	1.885	2.260	3.35	4.42	5.52	6.60	8.75
1:100:1/10	1.02	1.13	1.40	1.700	2.010	2.69	3.16	3.49	3.65	3.68
1/13	1.02	1.13	1.41	1.705	2.02	2.72	3.20	3.52	3.71	3.82
1	1.02	1.14	1.43	1.715	2.05	2.75	3.28	3.65	3.90	4.10
3	1.02	1.14	1.45	1.730	2.08	2.83	3.42	3.90	4.23	4.60
10	1.02	1.15	1.47	1.78	2.14	3.01	3.81	4.53	5.17	6.02

first parts of three-layer curves can always be made with the two-layer cases. This is illustrated by the curve labeled 3 in Fig. 4. It is a two-layer curve and may be seen to fit the other members of the family fairly well for values on the abscissa less than 3. This is definitely not generally true for the second portions of the curves. All the curves on Fig. 2 may be approximated by two two-layer curves but none of those on Fig. 4 can be analyzed in this manner. The second part of the curves are not close enough to asymptotic values in this

latter case to allow the two-layer approximation to be applied. To illustrate this point, the curve marked 2:2 on Fig. 5 will approach the curve marked Asy. for considerably larger values of the abscissa than are illustrated here. To attempt to fit 2:2 with a two-layer curve would involve a four hundred per cent error in the determination of the second interface depth. A rule with few exceptions is that three-

TABLE III

LISTING VALUES OF ρ_a FOR $h_1=4$, $h_2=8$

$\rho_1:\rho_2:\rho_3$ \ $8a/h_2$	2	4	6	8	10	15	20	25	30	40
1:1/100:1/10	.932	.690	.431	.246	.138	.056	.0496	.0540	.0588	.0665
1/3	.932	.691	.434	.250	.145	.0665	.068	.077	.0875	.105
1	.932	.691	.435	.252	.150	.0720	.0775	.090	.103	.130
3	.932	.691	.436	.253	.152	.0742	.0810	.0960	.112	.143
10	.932	.691	.436	.254	.155	.0748	.0823	.0978	.114	.147
1:1/10:1/100	.925	.719	.480	.293	.176	.0670	.0345	.0213	.0152	.0111
1/3	.945	.753	.543	.398	.310	.236	.239	.261	.278	.301
1	.950	.760	.560	.422	.350	.342	.400	.458	.510	.600
3	.955	.767	.574	.455	.393	.415	.510	.618	.720	.910
10	.958	.772	.584	.472	.422	.455	.575	.720	.858	1.12
100	.959	.775	.588	.481	.436	.475	.618	.765	.920	1.24
1:1/3:1/100	.920	.765	.575	.405	.280	.111	.0480	.0242	.0162	.0108
1/10	.935	.790	.621	.473	.369	.210	.154	.134	.122	.111
1	.966	.853	.726	.674	.660	.700	.745	.795	.830	.890
3	.972	.877	.785	.780	.816	1.02	1.21	1.37	1.51	1.74
10	.976	.888	.810	.815	.875	1.20	1.53	1.84	2.13	2.68
100	.980	.900	.840	.850	.950	1.32	1.74	2.16	2.60	3.45
1:3:1/100	1.020	1.100	1.170	1.170	1.115	.820	.455	.250	.148	—
1/10	1.02	1.11	1.210	1.230	1.175	.905	.505	.385	.262	.140
1/3	1.03	1.130	1.230	1.280	1.250	1.03	.800	.610	.495	.390
1	1.04	1.165	1.320	1.430	1.475	1.42	1.31	1.21	1.15	1.10
10	1.05	1.30	1.63	1.91	2.21	2.97	3.62	4.20	4.70	5.50
100	1.06	1.41	1.79	2.22	2.69	3.80	4.95	6.15	7.30	9.60
1:10:1/100	1.050	1.290	1.580	1.835	1.980	1.94	1.71	1.42	1.11	.565
1/10	1.06	1.30	1.60	1.85	1.99	1.99	1.78	1.51	1.23	.720
1/3	1.06	1.31	1.62	1.88	2.00	2.06	1.88	1.64	1.36	.860
1	1.07	1.315	1.65	1.965	2.125	2.31	2.27	2.12	1.92	1.43
3	1.07	1.34	1.73	2.075	2.34	2.84	3.16	3.35	3.41	3.30
100	1.090	1.43	1.92	2.45	3.03	4.50	5.95	7.40	8.80	11.6
1:100:1/10	1.04	1.40	1.91	2.47	3.00	4.14	5.00	5.62	6.00	6.40
1/3	1.04	1.40	1.91	2.47	3.00	4.14	5.00	5.62	6.05	6.50
1	1.04	1.40	1.91	2.48	3.01	4.19	5.08	5.72	6.20	6.60
3	1.05	1.43	1.92	2.50	3.02	4.25	5.25	6.00	6.55	7.05
10	1.06	1.44	1.96	2.60	3.15	4.50	5.70	6.70	7.55	8.70

layer curves containing a minimum may be approximated by two two-layer solutions with fair accuracy. Those containing a maximum cannot generally be so approximated within feasible electrode separations. This of course applies to analytical as well as graphical approximations. d) Fig. 5 illustrates a second point which might lead to incorrect construction of three-layer curves. The mathematical expression for the apparent resistivity of three-layers will reduce continuously to the expression for two layers if either h_1 or d is allowed to

approach zero. If in the case shown in Fig. 5, h_1 is allowed to vanish, curve 0:4 is approached. If d vanishes the curve marked 4:0 is the result. It is important to note that the resistivity curves for the limiting two-layer cases are *not* envelopes for the families of three-layer curves; a fact contrary to what might normally be expected. It is possible for members of these multiple-layer resistivity families to

TABLE IV

LISTING VALUES OF ρ_a FOR $h_1=2$, $h_2=8$

$\rho_1:\rho_2:\rho_3$ \ $8a/h_2$	2	4	6	8	10	15	20	25	30	40
1:1/100:1/10	.690	.252	.088	.040	.0295	.0311	.0362	.0410	.0459	.0545
1/3	.691	.259	.088	.047	.0340	.0368	.0442	.0529	.0610	.0775
1	.692	.262	.102	.052	.0365	.0388	.0475	.0575	.0670	.0870
3	.693	.264	.105	.054	.0380	.0400	.0495	.0605	.0715	.0930
10	.693	.265	.106	.055	.0385	.0405	.0502	.0612	.0725	.0960
1:1/10:1/100	.725	.325	.166	.102	.0695	.0348	.0220	.0160	.0130	.0102
1/3	.736	.350	.206	.162	.156	.176	.202	.225	.242	.267
1	.740	.361	.229	.192	.196	.250	.314	.370	.422	.505
3	.744	.370	.245	.219	.228	.296	.380	.462	.540	.685
10	.746	.376	.251	.227	.238	.322	.421	.525	.630	.825
100	.748	.378	.253	.232	.245	.329	.432	.540	.655	.880
1:1/3:1/100	.805	.508	.335	.233	.166	.068	.030	.0185	.0140	.0107
1/10	.820	.540	.381	.290	.235	.166	.138	.124	.115	.106
1	.840	.570	.490	.484	.500	.565	.640	.705	.758	.820
3	.848	.590	.533	.553	.605	.780	.960	1.12	1.26	1.51
10	.853	.600	.545	.581	.661	.920	1.19	1.44	1.69	2.14
100	.855	.610	.560	.610	.703	1.03	1.37	1.70	2.03	2.67
1:3:1/100	1.185	1.510	1.625	1.585	1.46	.970	.555	.325	.205	.100
1/10	1.190	1.520	1.635	1.608	1.493	1.04	.650	.420	.283	.152
1/3	1.195	1.540	1.680	1.695	1.620	1.28	.890	.650	.515	.400
1	1.200	1.570	1.770	1.870	1.895	1.75	1.55	1.38	1.27	1.14
10	1.210	1.630	2.130	2.530	2.92	3.78	4.46	5.10	5.60	6.39
100	1.235	1.730	2.33	2.95	3.55	5.10	6.60	8.15	9.63	12.6
1:10:1/100	1.36	2.125	2.72	3.070	3.135	2.97	2.53	1.93	1.32	.608
1/10	1.36	2.13	2.74	3.09	3.23	3.03	2.57	1.96	1.42	.760
1/3	1.37	2.15	2.77	3.11	3.26	3.19	2.75	2.18	1.64	.950
1	1.37	2.16	2.80	3.18	3.45	3.54	3.25	2.74	2.23	1.48
3	1.38	2.19	2.94	3.45	3.76	4.10	4.09	4.00	3.87	3.64
100	1.42	2.34	3.31	4.32	5.39	7.75	10.0	12.2	14.2	18.0
1:100:1/10	1.41	2.55	3.65	4.68	5.65	7.60	9.00	9.90	10.4	10.8
1/3	1.41	2.55	3.67	4.70	5.70	7.70	9.10	10.0	10.5	11.1
1	1.42	2.57	3.70	4.75	5.72	7.85	9.43	10.5	11.2	11.8
3	1.42	2.59	3.76	4.82	5.85	8.15	10.2	11.9	13.2	15.1
10	1.47	2.65	3.86	5.00	6.18	8.80	11.3	13.5	15.2	17.7

cross. This fact makes the task of extrapolation of curve families difficult.

The interpretation method proposed is simply that of matching field curves with a set of standards plotted from the data given in the accompanying tables. With sufficiently closely-spaced standard curves this method is theoretically capable of yielding a unique answer for the resistivity function. A proof of the uniqueness theorem implicitly involved in this procedure has been given.[2]

To use the standard data most effectively they should be plotted on logarithmic graph paper. This makes the shape and size of the curves independent of the units of resistivity and electrode separation used. The effect of plotting to different field units is obviously only

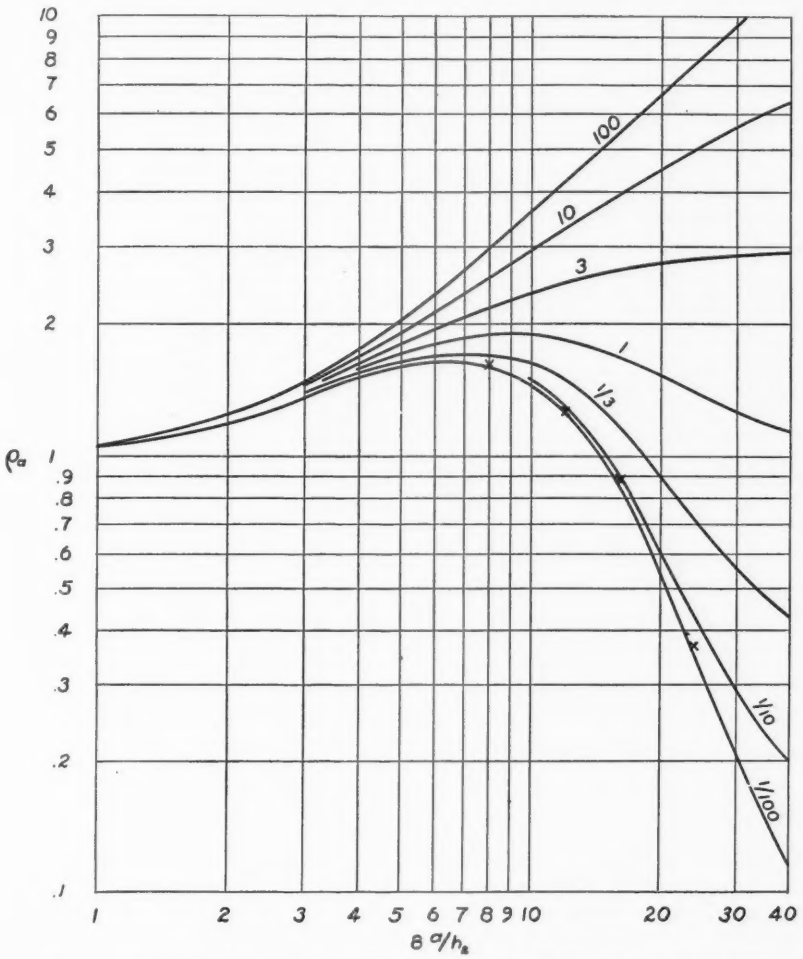

FIG. 4. Illustrating the effect on the apparent resistivity of variations in ρ_3 all other factors remaining constant. $h_1:d$ as 1:3 and $\rho_1=1$, $\rho_2=3$, The labels on the curves give the values of ρ_3.

that of shifting the curve bodily but not distorting it. Field curves plotted on the same paper may then be compared with the standards

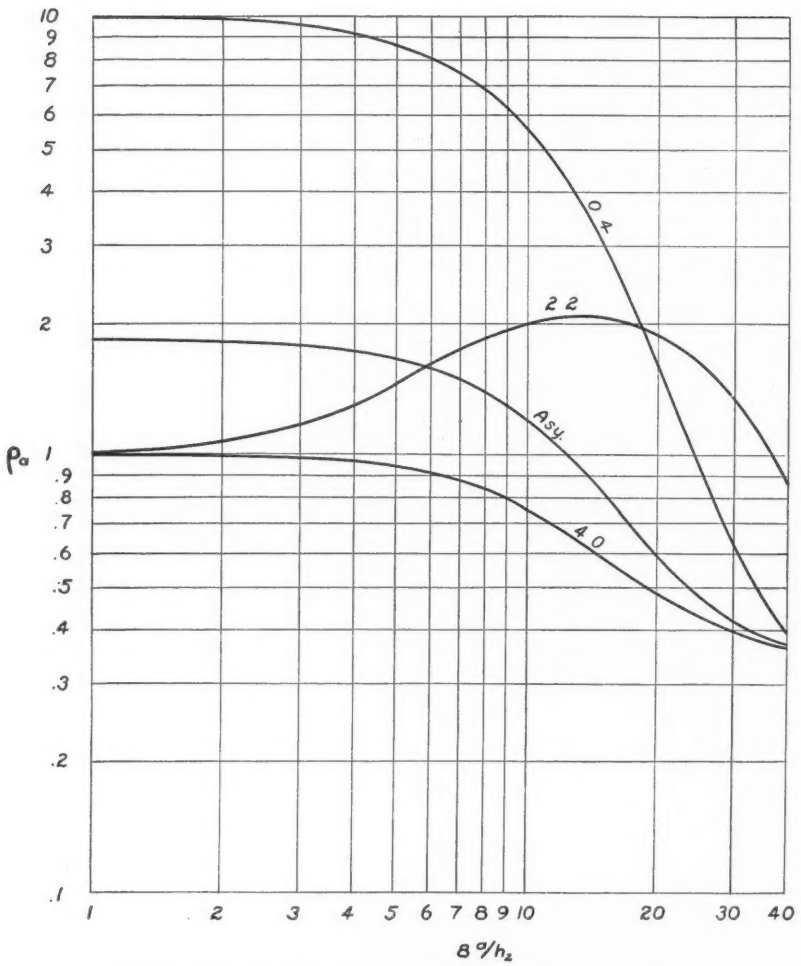

FIG. 5. Illustrating the behavior of a certain type of resistivity curve. The numbers on the curves give the relative thickness of the first and second layers. The curve marked Asy. is the-two layer case obtained by applying Kirchhoff's law to the two upper layers according to Hummel's method.[4] Curve 2:2 will approach this curve for large electrode separations. Within the region of the calculation this asymptotic value cannot be used as a guide in an analysis of 2:2. The figure further illustrates the fact that the three layer curves need not lie within the curves for the limiting two-layer cases. Curves 0:4 represents the apparent resistivity variation for a two-layer case with $\rho_1=10$, $\rho_2=\frac{1}{3}$ and the interface at 8. 2:2 represents the apparent resistivity variation for a three-layer case where $\rho_1=1$, $\rho_2=10$, $\rho_3=\frac{1}{3}$ and $h_1=4$, $d=4$ units. 4:0 represents the apparent resistivity variation for a two-layer case where $\rho_1=1$, $\rho_2=\frac{1}{3}$ and the interface is at 8.

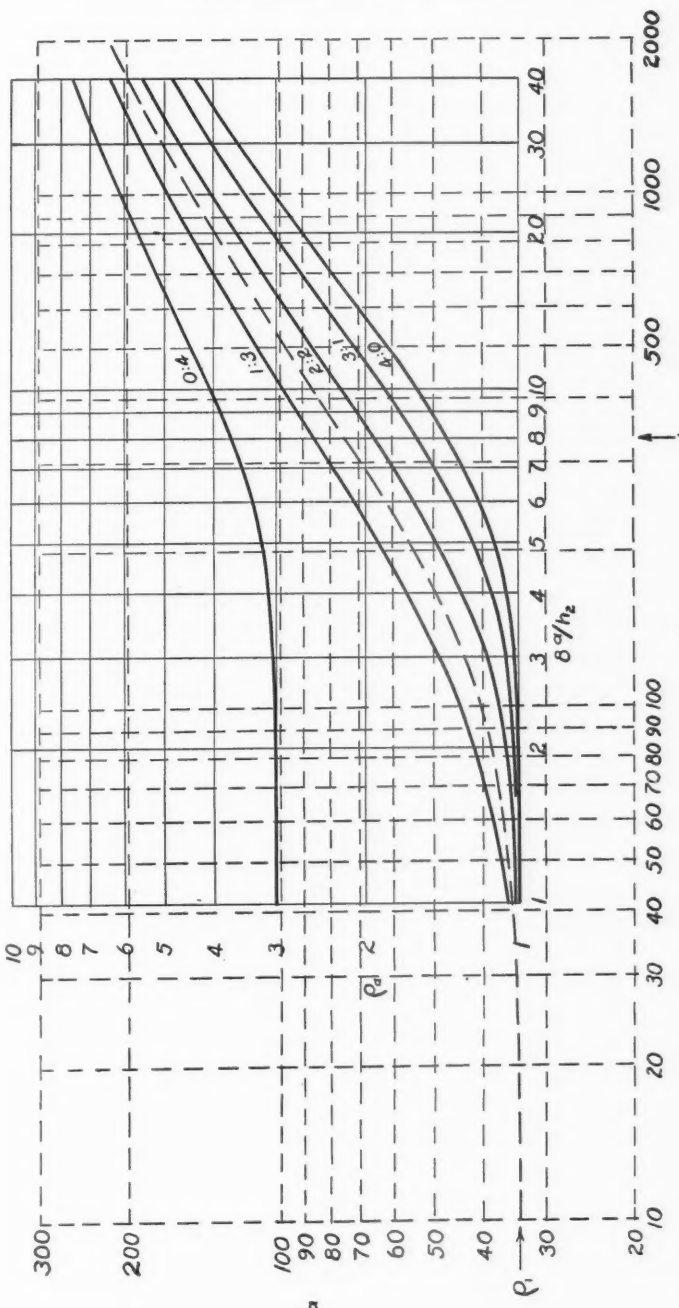

FIG. 6. Shows the superposition of a field curve on a set of standard curves. The standards have the values of $\rho_1:\rho_2:\rho_3$ as 1:3:10. h_2 is 8 units and the labels on the standard curves give the value of $h_1:d$. For the explanation of the interpretation procedure see the body of the text.

by superposition.[5] Families of standards having h_1 as the variable are most useful. (See Fig. 2.) When a fit has been obtained the problem is essentially solved by reading certain constants from the curves as is illustrated in Fig. 6. Here the dashed plot represents the field sheet with its resistivity curve plotted in any convenient units and the solid plot represents the standard family which by trial has been found to contain the field curve. The standard family represents curves for $\rho_1:\rho_2:\rho_3$ as 1:3:10. On the standard sheet the second interface occurs at a depth equal to an electrode separation $a=h_2$, or at the abscissa point $8a/h_2=8$, as marked. Three readings of constants are necessary. From the ordinate on the field data opposite 1 on the standard ordinate we read $\rho_1=33$ ohm cm., ohm feet, or whatever unit has been used to express the measured ρ_a. On the abscissa below the point 8 on the standard one reads $h_2=330$ feet, meters, yards or whatever unit has been chosen for plotting the measured electrode separation. Finally the $h_1:d$ ratio is read by noting that the field curve lies about half way between the standards whose ratios are 2:2 and 1:3. Therefore the $h_1:d$ ratio for the field curve corresponds to about 3:5. From these facts we at once find $\rho_1=33$, $\rho_2=100$, $\rho_3=330$, $h_1=124$ and $h_2=330$ field units.

The advantages of this interpretation method are rather obvious. It obtains the field constants with the maximum possible accuracy since it uses the entire resistivity curve to evaluate them. It is simple and rapid in application. The plotting system which renders the shape of the curves independent of the field units has an advantage for the interpreter in allowing him to become familiar with curve trends. This is difficult to do if linear plots are used. The use of the method will emphasize the necessity of running the field curves to large electrode separations since it will be obvious to the interpreter that unique answers cannot otherwise be obtained. It will also emphasize the necessity for care in obtaining the ρ_a for small electrode separations.

The limitations of the method are equally obvious. The curves can be applied only to a three-layer earth where good approximations to homogeneity of materials and uniform layer thickness over relatively large areas exist. The method does not eliminate but only re-

[5] A convenient paper for this process is sold by Keuffel and Esser Co. Their No. 358-112L Logarithmic 3×2 cycle is printed on heavy stock and will be found suitable for the standard curves. No. 359-112L is on tracing weight stock, printed to the same scale and can be used for the field curves. No light table is then necessary for comparing the two sheets.

duces the difficulty of obtaining the surface resistivity correctly. In place of one determination of the surface resistivity the whole first portion of the field curve is implicitely involved but the interpretations are sensitive to variations of the curve in this region. Further, in certain of the curve families of which 2:2 of Fig. 5 is an example, the extrapolation of the families to members representing thin upper layers is an uncertain matter. Finally it is not claimed that this interpretation method may be used automatically to grind out answers. Judgment and experience in the application of resistivity methods are still required in order to obtain even the optimum 10 per cent accuracy.

In conclusion the authors wish to express their thanks to Prof. L. B. Slichter for his continued interest in the solution of this problem and for the advice he so frequently gave during the course of the work.

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MAPPING FAULTS BY THE REFLECTION METHOD*

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ABSTRACT

In regions where reflections can be correlated from point to point the mapping of faults presents little difficulty, and the hade as well as the throw of a fault can be generally determined. Two records secured across a fault are reproduced and a cross section derived from these records is shown. A description is given of a method of profiling which permits correlation of records in areas where the ordinary correlation method cannot be successfully applied. The application of this method in the determination of faults is outlined.

INTRODUCTION

The reflection seismograph method was first successfully employed in the Seminole Plateau of Oklahoma where it was applied strictly as a correlation method. In this region certain limestone beds are laterally persistent for considerable distances and reflections from these beds were readily identified over widely spaced points. In general, a correlation of the reflections presented little difficulty and large areas were mapped by this method.

When the reflection method was later introduced in the Gulf Coast, it was found that, due to the absence of persistent reflecting horizons, the correlation method could not be effectively employed. The dip method, therefore, was evolved and soon came into general use in the Gulf Coast, in California, and in other areas where the beds are not laterally persistent. The effectiveness of this method, in mapping uplifts and structural folds, is now generally recognized, and many outstanding discoveries can be credited to its use. However, as commonly employed, it is not effective in the mapping of faults, and this constitutes its chief limitation.

The purpose of this paper is to outline briefly the general application of the reflection method to the mapping of faults, to describe a method of continuous correlation, and particularly to show how this method can be applied to determine faults in those areas where the ordinary correlation method cannot be effectively employed.

MAPPING FAULTS BY CORRELATION

In areas where reflections from definite horizons can be correlated from point-to-point, the mapping of faults presents no unusual difficul-

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ty. Any unusually abrupt change in the depth of the reflection beds is a possible indication of a fault. By securing reflections at closely spaced points along a line crossing the indicated region of faulting, the position and the amount of displacement of a fault can usually be determined.

The hade of a fault can be established by correlation only if the points of faulting, on two or more reflecting beds appreciably spaced in the section, are determined within fairly close limits. This can be frequently accomplished by securing reflections at closely spaced points across the fault. However, if an appreciable gap exists over which satisfactory reflections cannot be obtained, it may not be possible to determine its hade although the approximate position and displacement of a fault can always be established from a correlation of the reflections.

An interesting case of a fault located from a single record on which were recorded reflections from both sides of a fault plane is indicated on the reflection cross-section diagram shown in Figure 1. On this cross section are also shown the disposition of the seismometers between the two shot-points designated 419 and 420 as well as the paths of the reflected waves from one of the reflecting beds. The total spread of the seismometers is 1,000 feet, with the first and last seismometers located 150 feet from the adjoining shot-points. Twenty-four seismometers were used and placed in two groups of twelve each with a gap of 700 feet between the 12th and 13th seismometers. The first group of seismometers actuated the first five traces of the records while the second group actuated the last five traces.

Records were secured from both shot-points without any change in position of the seismometers. As may be seen from the cross section, the fault was crossed between traces 5 and 6 of the records from shot-point 419. Therefore, separate determinations of depth were made for the first five and the last five traces of these records and these depth determinations have been plotted on the cross section.

The reflections on the records from shot-point 420 indicate no faulting since there is no abnormal change in the reflection arrival times on the various traces. Hence it was possible to compute both the depth and dip of the various beds. The dip determinations showed the beds to be absolutely flat and they were so plotted on the cross section.

Figure 2 shows the two records from which the cross section shown in Figure 1 was plotted. In this figure the reflections corresponding to

the beds shown on the cross section by the letters H, I, J, K and L are similarly designated on both records. From an inspection of these

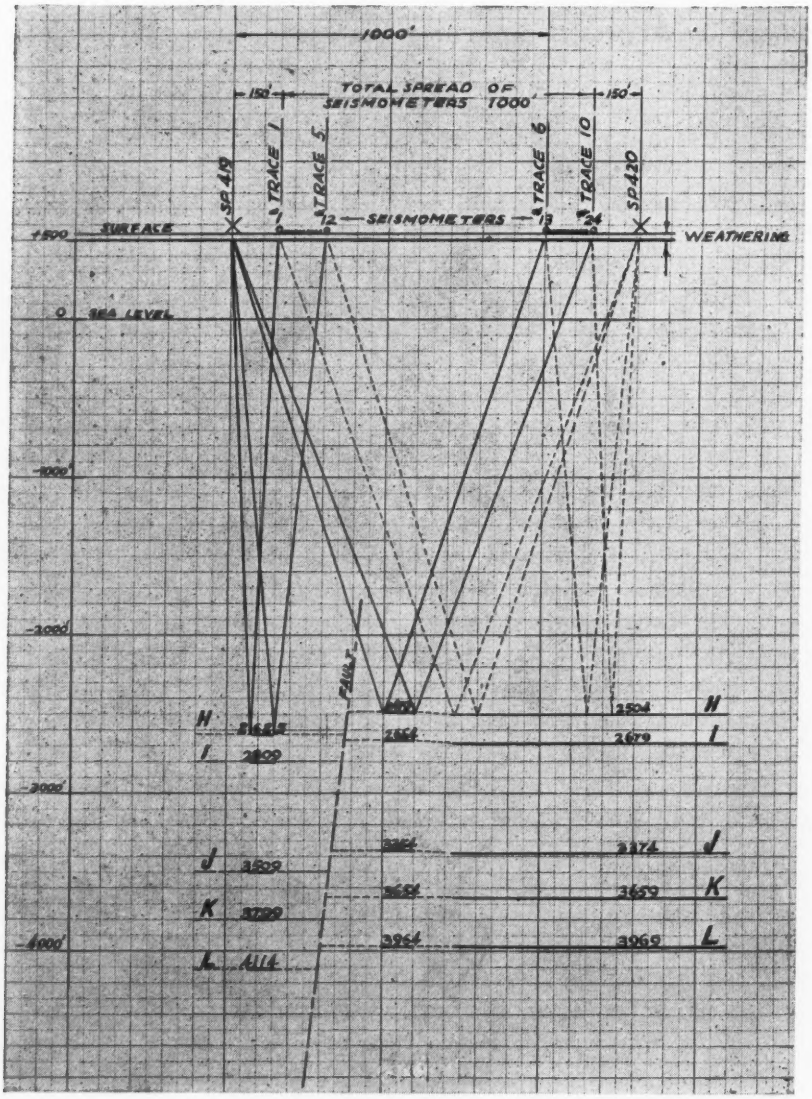

FIG. 1

records it may be seen that the reflections are quite clear and distinctive and that the correlation is apparent. The fault is clearly indicated

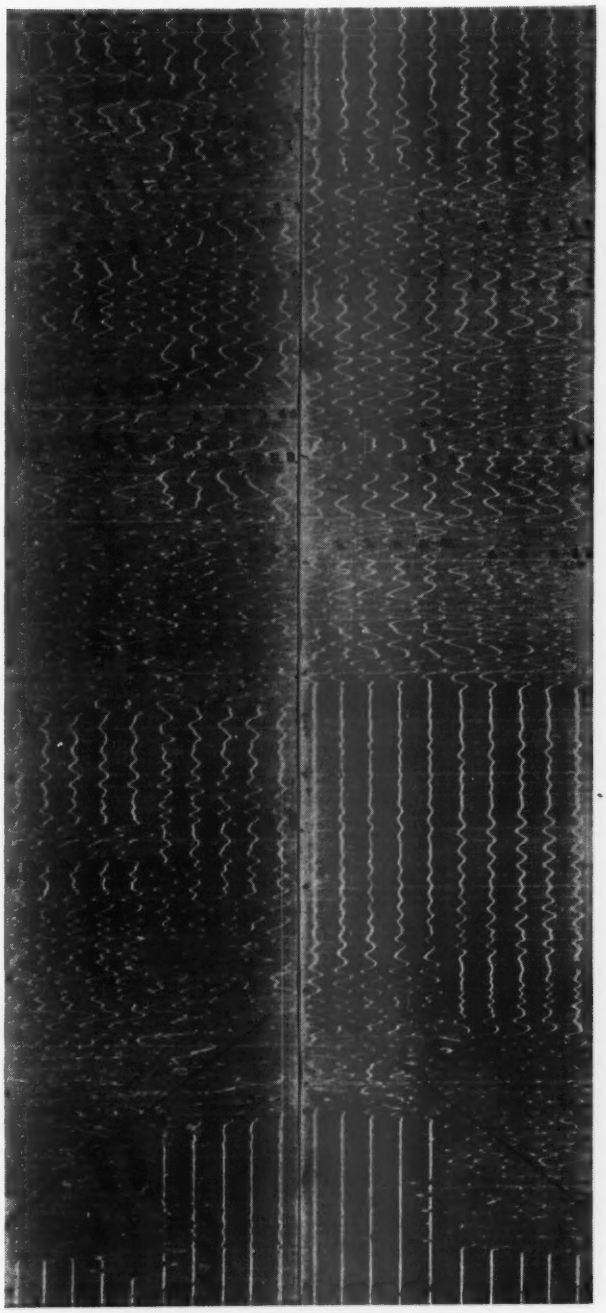

FIG. 2

on the record from shot-point 419 by the sudden and substantial decrease between the arrival times of the reflections on trace 5 and those on trace 6. As the seismometers actuating trace 6 are 700 feet farther from the shot-point than those actuating trace 5, the arrival times of the reflections on trace 6 would be greater, under normal conditions, than those on trace 5. Since the reverse is true, and since the difference in time is entirely too great to be accounted for by dip, faulting is definitely indicated.

Figure 3 shows an enlarged portion of the record from shot-point 419 shown in Figure 2. It is clearly seen from this figure that the character of the reflections is identical on all ten traces, and there can be little doubt as to the correctness of the correlation of those traces from which faulting is determined, namely traces 5 and 6.

It is of interest to note that in the case of this fault there is no change of consequence in the amplitude and character of the reflections as the paths cross the fault plane. Even if it is assumed that the fault does not extend higher in the section than bed J, it may be seen from Figure 1 that the paths of the reflections on the last five traces of the record from shot-point 419 cross the fault plane while those on the first five traces do not. The record shown in Figure 3, therefore, affords a good comparison between reflections whose paths have crossed the fault plane and those which have not, and shows definitely that in this case, the fault plane had no appreciable effect on the transmission of wave energy.

CONTINUOUS CORRELATION METHOD

In areas where beds are not laterally persistent, such as parts of the Gulf Coast and California, correlation of reflections between shot-points is exceedingly difficult and oftentimes impossible. In such areas the dip method of survey is generally employed. Dip determinations are made at spaced points along lines of traverse and by extending the dips half way between adjacent shot-points, a map of the subsurface is derived. Where no faults are present, excellent results may be obtained by this method as dip determinations can now be made with an accuracy of better than one degree. However, in faulted regions the effectiveness of this method is very limited as dip determinations at various points seldom afford definite recognition of a fault, and in no case permit the determination of its throw.

Dip determinations provide direct evidence of faulting only in those cases where reflections can be secured from a fault plane. Such

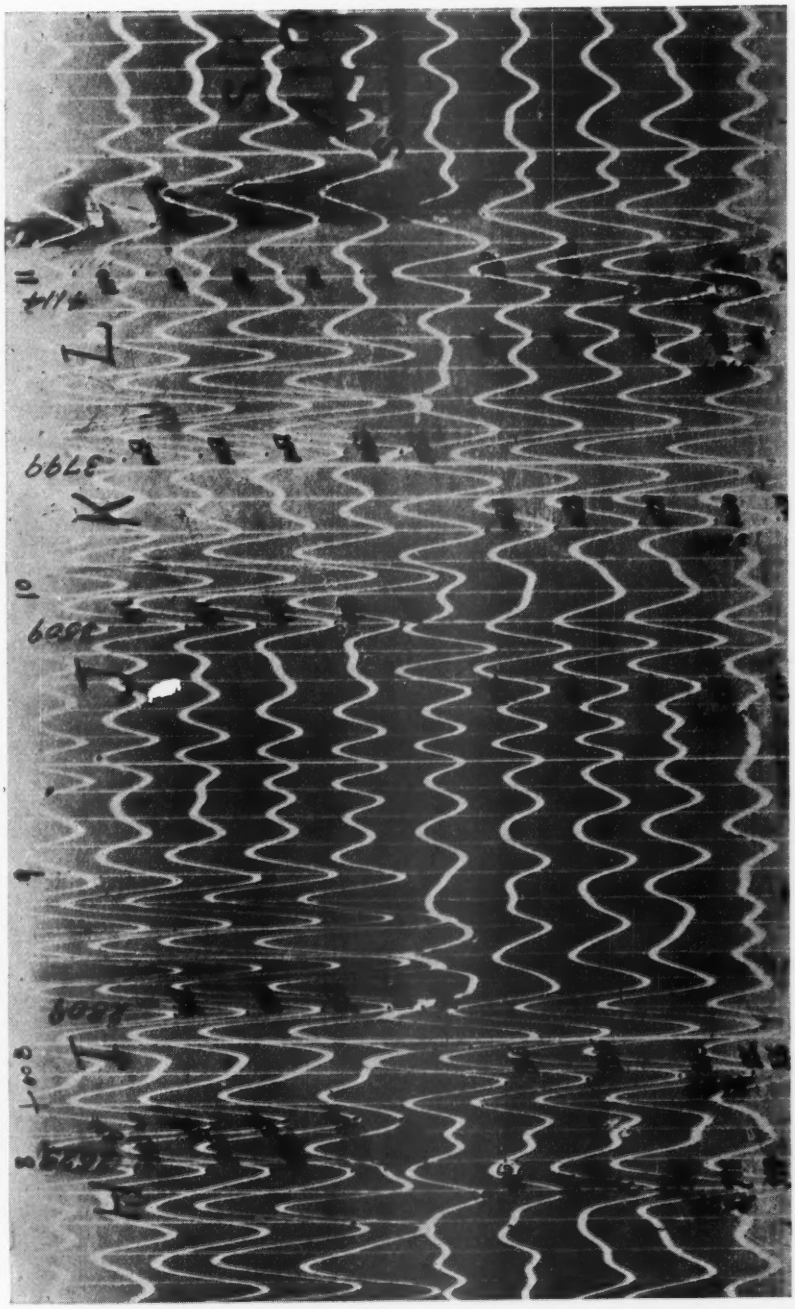

FIG. 3

reflections are recognized from their abnormal "step outs" as compared to those of the other reflections. The position and hade of the fault may then be established by determining the depth and dip, but its throw cannot be determined except by correlation.

However, faulting can be established only occasionally in this manner as reflections from a fault plane cannot often be secured. It is obvious that wave energy will be reflected from a fault plane only at points where two beds with different physical characteristics come into sharp contact. Especially in the case of a fault with a relatively small displacement there will be many portions of the fault plane along which either the same beds or different beds of approximately similar characteristics are in contact. Consequently, even if the plane of faulting is very sharp it will seldom act as a good reflector of wave energy throughout any large portion of its vertical dimension. For this reason, reflections from a fault plane are not frequently recorded.

With the exception of the infrequent cases where reflections are secured from a fault plane, direct evidence of faulting can be obtained with the reflection method in only two ways; first, by effecting a correlation of reflections from opposite sides of a fault, and second, by observing the break in the continuity of all the beds which must occur along a fault plane. Since the dip method makes use of only those reflections which are continuous throughout all the traces of the individual records, and since it does not involve correlation of records secured at different points, it follows that its effectiveness in mapping faults is greatly limited.

A dip determination, of course, also involves correlation. But instead of a correlation of records from spaced points, it involves a correlation of reflections which appear on the various traces of a single record. Because this can be effected with much more certainty than a correlation of records from spaced shot-points, dip determinations can be made in many areas where point-to-point correlation is impossible. Therefore, in order to effect a correlation in such areas, it is necessary only to perform continuously that which is performed for a comparatively short distance in a dip determination. This not only requires a method of continuous profiling, but also one which affords positive correlation of records from adjacent shot-points.

Figure 4 shows an arrangement of the seismometers and shot-points which will permit a continuous correlation of records in any

locality where several reflections can be secured on each record. The seismometers for each spread are laid out opposite two adjacent shot-points so that the first and last seismometer is each opposite one of the shot-points.

Records are secured from shots fired at shot-point A and then, without disturbing the seismometers, from shots fired at shot-point B. The seismometers are next moved and placed in the same manner opposite shot-points B and C and records are secured from shots fired at these shot-points. This process is then continued for as many shot-points as desired.

The records secured in this manner are similar to those in making dip determinations and the reflections, therefore, can be followed from trace to trace with no more difficulty than is experienced on the usual dip record.

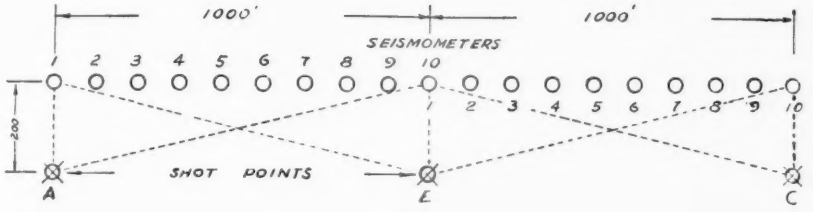

FIG 4

It is apparent that with this arrangement of seismometers and shot-points, each reflection arriving at seismometer #10 from shot-point A has the same length of path and almost identical point of reflection on the bed as the corresponding reflection arriving at seismometer #1 from shot-point B. Their reflection times, therefore, must be equal except for any difference in the thickness of the weathered layer at seismometers #1 and #10. Hence, a correlation of the records from shot-point A and shot-point B can be made simply by matching reflections of equal times on the two corresponding traces. In this manner each reflection which persists on all the traces of both records can be correlated from trace #1 of the record from shot-point A to the trace corresponding to seismometer #10 from shot-point B.

In order to extend the correlation from shot-point B to shot-point C, a record from shot-point B obtained with the seismometer spread opposite A and B must next be correlated with one secured with the seismometer spread opposite B and C. Since the seismometer directly

opposite B is common to both spreads, the two records have a common trace and, therefore, can be readily correlated. A correlation of the records from B and C is then made in the same manner as between A and B. Thus a correlation has been effected from A to C and may be extended to as many successive shot-points as desired.

It is seen that in this manner a continuous correlation of records from consecutive shot-points may be effected with no more difficulty than is experienced in making dip determinations in the same locality. Of course, while this method permits a continuous correlation of records, it does not necessarily insure a continuous correlation on the same beds throughout the region. If a bed pinches out, reflections from this bed naturally will not be recorded beyond the point of pinch-out, and the correlation on this particular bed will end at this point. However, as experience with dip shooting has shown, while some reflections cease to appear, others replace them, so that in any locality the number of reflections on each record will remain roughly the same from one shot-point to another.

Consequently, if the records from two adjacent shot-points have several reflections which appear on all the traces, it will almost always be possible to correlate at least a few of the reflections on both records. If a fault is crossed, the continuity of all the reflections may be broken, but as will be shown in what follows, the application of this method greatly facilitates the determination of faults in those regions where the usual point-to-point correlation is impossible.

APPLICATION OF CONTINUOUS CORRELATION METHOD IN FAULTED AREAS

Let us now consider the application of the continuous correlation method just described in shooting across faults. We shall first take up the case of a vertical fault with a sharp fault plane. In running a line of continuous correlation profiles across such a fault, a record would be secured on which all the reflections would have an abrupt change in arrival times between the traces disposed on opposite sides of the fault. Since, in general, there would be many reflections on the record, considerable confusion would result due to the fact that some reflections from different horizons on the up and down-throw side of the fault would, as a rule, fall approximately in line with each other. However, as the records from adjacent points would indicate the usual continuity of most of the reflections, the abrupt change which would occur on all the reflections of the record would necessarily imply a

corresponding break in the continuity of all the beds, and this is certainly good evidence of faulting. Having thus recognized the fault, those traces between which the abrupt change takes place are then more closely scrutinized, and this will often reveal a fairly good correlation on at least one or two of the more characteristic reflections. From this, the displacement of the fault can be determined. If a correlation of the traces cannot be made, nothing can be learned of the throw of the fault although its position can always be fixed from the disposition of the seismometers.

In the case of a normal fault with an appreciable hade, the problem of direct correlation of reflections from the up-throw and down-throw sides is much more difficult and oftentimes impossible. Unlike the case of a vertical fault, the break in the continuity of all the reflections does not occur between approximately the same traces of a single record. Due to the hade of the fault, the breaks in continuity of the various reflections will occur between different traces and sometimes on different records, depending on the point where the fault plane crosses each particular reflecting bed. Since there is no uniform break in the continuity of all the reflections of a single record, the existence of faulting is not readily recognized as the break in continuity which occurs on only few of the reflections may be easily overlooked.

Let us consider a fault such as is shown in Figure 5 having a sharp fault plane, a hade of 30° and a throw of 600 feet. For the sake of simplicity it may be assumed that only six reflecting beds are present, and that all the beds are flat. As shown in the figure, the seismometers are laid out opposite the shot-points in accordance with the scheme previously described.

Now let us consider the records which may be obtained from a seismometer spread opposite shot-point X and Y. The records from shot-point X will show reflections from all the beds except bed F. A reflection from this bed will appear only on the first six traces as the points of reflection for the last four traces will lie in the fault plane and no energy from these points will be returned to the seismometers actuating these traces. Since only reflections which carry through on all traces are ordinarily picked, the continuity of this bed will end at this point, and this would probably not be attributed to faulting.

All the traces of the records from shot-point Y will show reflections from beds A, B and C. Reflections from beds D, E and F will appear only on some of the traces and, therefore, will not be picked.

Since the upper three reflections carry through on all the traces, no special significance might be attached to the break in continuity of these three lower beds and faulting might be overlooked.

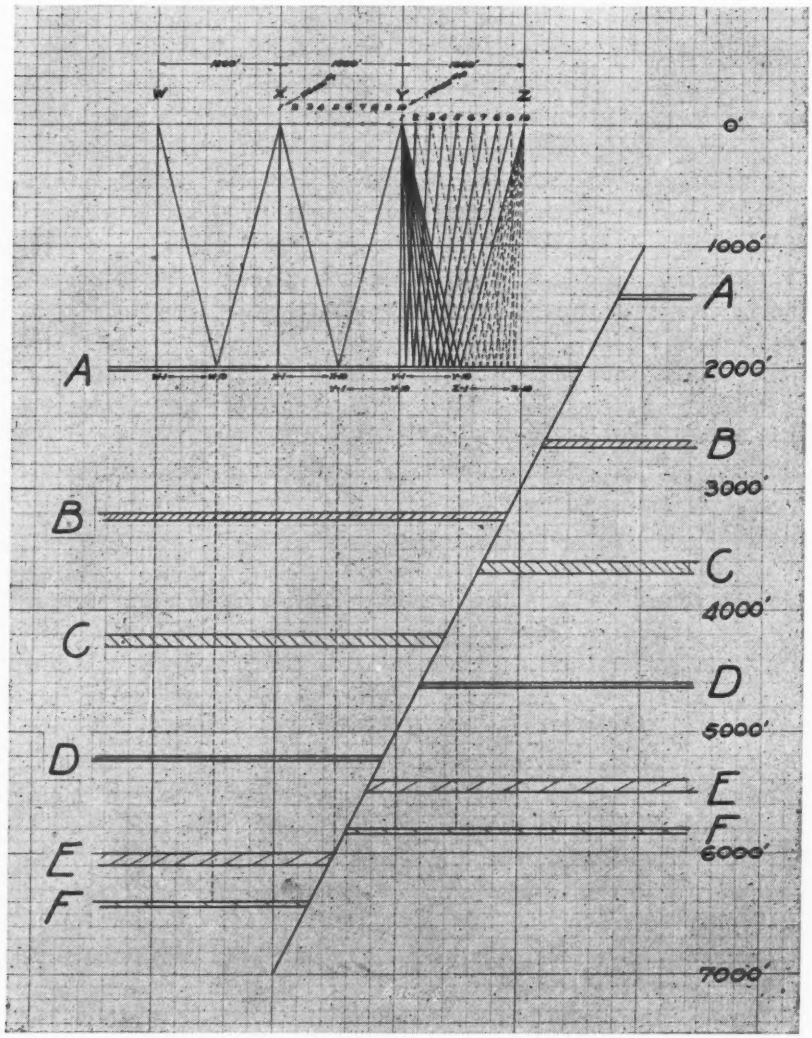

FIG. 5

If we now consider the records from a seismometer spread opposite shot-points Y and Z, we will find a similar condition, except that

the break in continuity will occur on the reflections higher up in the section. Records from shot-point Y will show persistent reflections from all the beds except beds C and D. Reflections from these two beds will not appear on all the traces, but this might be readily attributed to ordinary lensing of beds rather than to the presence of a fault. Similarly, records from shot-point Z will have persistent reflections from all the beds except beds B and C, and here again the break in the continuity of the reflections from these two beds might not be recognized as an indication of faulting.

It is seen, therefore, that in areas where the pinching out of beds is a normal expectation, a normal fault with an appreciable hade cannot be readily recognized merely from an inspection of the records. Since records from each profile will, in general, have several reflections which appear on all traces, any break in the continuity of a few of the reflections may not be considered significant. *It is due chiefly to this fact that dip profiles across faults will often reveal no indication of faulting.*

But if the results are plotted on a cross section, the fault will be readily recognized. By plotting only the reflections which carry through on all traces a cross section of the case which we have just considered would appear as shown in Figure 6. It is apparent from this cross section that a fault would be readily recognized along the zone throughout which no reflections carry through. Having this indication, a closer scrutiny of the records is made, and in some cases, some sort of correlation can be affected. Even if no reliable correlation is possible, such alignment of the breaks in the continuity of the various beds would permit little doubt as to the existence of a fault.

The gap in the continuity of reflections which always occurs in the case of a normal fault will, in most cases, preclude the possibility of effecting a correlation across the fault on the traces of a single record. Consequently, the only correlation possible is between records from separate points, and this, in the areas we are considering, may be impossible. In many cases, therefore, recognition of faulting can be gained only from an inspection of a cross section plotted from records obtained in accordance with the continuous correlation method.

In the case of a thrust fault, the problem would be essentially the same except that the break in continuity would always occur between nearby traces. The possibility of effecting a correlation across the fault, however, would be only slightly better since the change in reflection arrival times between adjacent traces might be interpreted

as the pinching out of a bed. As a result, recognition of a thrust fault must be likewise generally acquired from an inspection of a cross

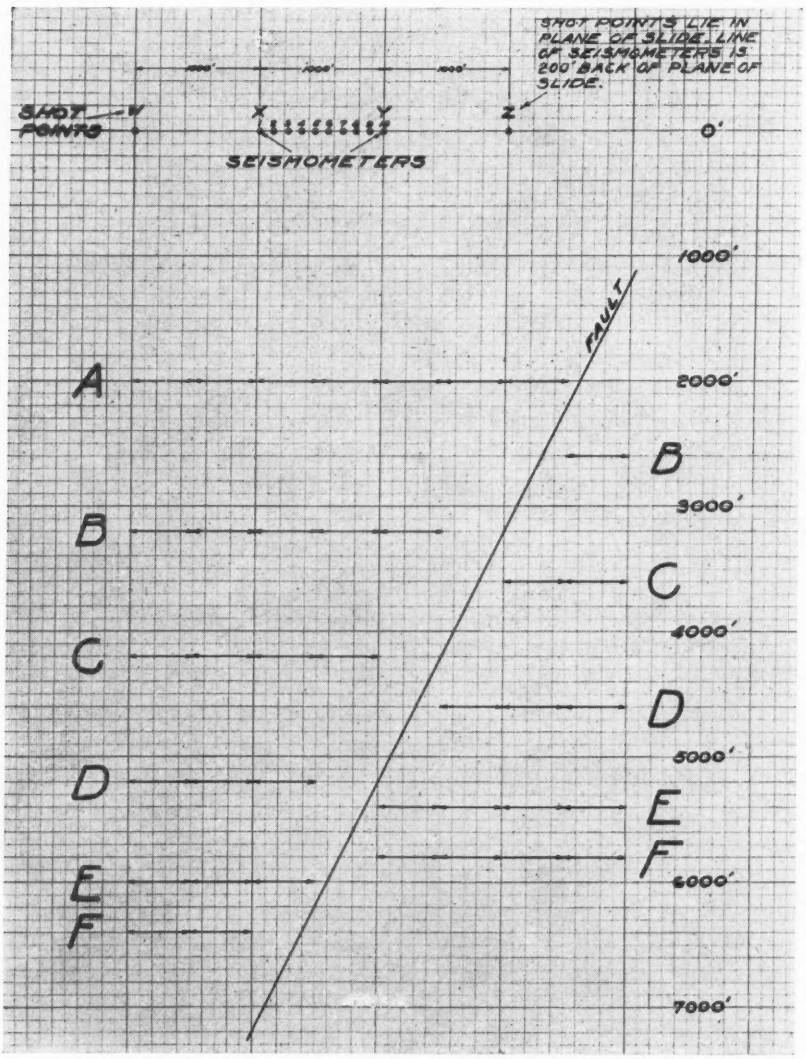

FIG. 6

section plotted from records obtained by the continuous correlation method.

Although we have been considering cases where the beds are flat

the same procedure can be applied equally well where the beds are inclined. In the latter case, it is apparent that the reflecting beds must be plotted on the cross section with their corresponding dips. As dips can be calculated in the usual manner for each profile, this introduces no difficulty in recognizing the fault from the cross section.

In the various types of faults we have so far considered, the fault planes were assumed to be very sharp. But a fault plane is not always sharp, it sometimes consisting of a zone of fracturing and faulting of appreciable width. This zone may extend from a few feet to several hundred and, in the case of a major fault, to several thousand feet. When shooting across such a fault, therefore, good reflections over the zone of faulting and fracturing are not to be expected.

The existence of such a zone does not, as a rule, increase the difficulty of determining the fault except where the zone is of such a width as to break the continuity of the reflections on several successive profiles. When this occurs, faulting may not always be easily recognized. Plotting the results on a cross section will show only a comparatively wide and poorly defined section where no consistent reflections have been obtained. Due to the fact that all the records secured across this zone will in general be poor, one is apt to conclude that this is due to some unfavorable surface condition associated with the locality. But such a conclusion should not be drawn too hastily. Since a gap with no consistent reflections is a logical expectation when shooting across a fault with a wide zone of fracturing, one should not conclude that there is something irregular with the locality unless there is definite surface evidence to support the conclusion. If the poor results cannot be attributed to some known cause, the existence of such a gap should always be considered as a possible indication of faulting.

A close study of the characteristics and continuity of the reflections is made, and from the results of this study some valid inferences can be usually drawn as to the possible existence of the faulting. If a possibility of faulting is indicated, a second line of profiles approximately parallel to the first, is then run, and it is noted whether the same results are obtained. If a gap of poor results is again encountered, the existence of a fault is indicated. By running additional parallel lines of profiles, the same indication may then be obtained at other points and thus the existence and direction of the fault can be established.

In the faults most frequently encountered, the zones of fracturing

do not exceed two or three hundred feet in width and consequently the break in continuity of all the reflections seldom occurs on the records of more than one or two profiles. The problem of recognizing such faults, therefore, differs only slightly from that of faults with sharp fault planes since the zone of poor results is comparatively narrow and well defined. A plot of the reflections on a cross section will readily reveal an alignment of the breaks in continuity of all the reflections, and while the zone of poor results will be somewhat broader, the fault will still be recognized.

It might be contended that, particularly in the case where the gap of poor reflections is of considerable width, such indirect evidence is hardly sufficient to establish a fault, and in some cases this may be true. Generally, however, one with skill and experience in interpreting continuous correlation records will not have much more difficulty in arriving at a final conclusion as to the existence of a fault than is had in arriving at a correlation of reflections in areas such as Kansas, for example, where the ordinary correlation method is almost universally employed. As in the case of many correlations, the final interpretation cannot always be reached without some mental reservations. But this will not necessarily mean that such faults cannot be generally located if the procedure above described is intelligently applied.

THE REFLECTION OF ELASTIC WAVES FROM TRANSITION LAYERS OF VARIABLE VELOCITY*

ALFRED WOLF†

ABSTRACT

Elastic waves are reflected not only from discontinuities in the medium in which they are propagated, but also from transition layers in which the elastic constants are continuous functions of position; the coefficient of reflection is then a function of wave length. Section 1 of this paper gives explicit formulae for the coefficient of reflection of continuous waves from such a layer, at vertical incidence. In Section 2 the manner of variation of the coefficient of reflection with the angle of incidence is discussed qualitatively. Finally, in Section 3, the shape of a pulse reflected from a transition layer is determined.

The reflection of elastic waves from discontinuities has been studied by many authors.[1] When the density is constant, the coefficient of reflection of longitudinal elastic waves at vertical incidence is given by the simple formula

$$R = \frac{k - 1}{k + 1}, \tag{1}$$

where k denotes the ratio of the velocities on the two sides of the surface of discontinuity. It is proposed to discuss a generalization of Equ. (1), which replaces it when the two regions with the velocities v and kv are separated by a transition layer in which the velocity is a continuous function of position. The discussion will be confined mainly to vertical incidence because the general case is much too complicated, and only some qualitative statements can be made about it.

Certain general conclusions can be stated at the outset. It is evident that the reflection coefficient will be a function of the wavelength of the incident elastic waves. When the wavelength is very long compared to the thickness of the transition layer, the presence of the latter is clearly of no importance, so that the coefficient of reflection is given by Equ. (1). Very short waves, on the other hand,

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[1] For literature cf. J. B. Macelwane: Theoretical Seismology I. Chapt. VII.

will not be reflected at all because there is no effective discontinuity in the medium in which they are propagated.[2]

1. THE COEFFICIENT OF REFLECTION FROM A TRANSITION LAYER AT VERTICAL INCIDENCE

Consider an elastic medium of constant density, in which a layer of thickness h provides a gradual transition between two regions A and B, in which the velocities of propagation of longitudinal elastic waves are respectively v and kv (Fig. 1). Let a coordinate system be introduced with its origin O at the boundary between region A and the transition layer. The x-coordinate is taken perpendicular to the latter, its positive direction being toward region B. The velocity of propagation in the transition layer C is assumed to be a linear function of x.

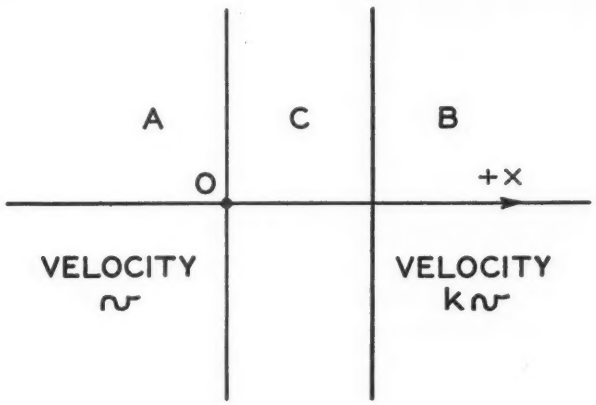

FIG. 1

Under these circumstances, the equation of propagation of longitudinal waves of amplitude u in the x-direction can be written

$$\frac{\partial}{\partial x}\left(V^2 \frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial t^2} \tag{2}$$

[2] While this paper was in the course of publication, the writer's attention was called to an article of Sezawa and Kanai in the Bulletin of the Earthquake Research Institute of the Tokio Imperial University, Vol. 13, 1935. Sezawa and Kanai cover approximately the same ground as the first section of this paper, and reach substantially the same conclusions. Their work is, however, based on a different law of variation of velocity with distance in the transition layer, so that an exact agreement cannot be expected.

where

$$
\begin{aligned}
V &= v && \text{for } x < 0 \\
V &= v\left(1 + \frac{k-1}{h}\, x\right) && \text{for } 0 < x < h \\
V &= kv && \text{for } h < x.
\end{aligned} \tag{3}
$$

The solutions of Equ. (2) for harmonic waves are

$$
\begin{aligned}
&\exp j\omega(t \pm x/v) \quad \text{for } x < 0 \\
&\exp j\omega(t \pm x/kv) \quad \text{for } x > h
\end{aligned} \tag{4}
$$

in the regions A and B respectively, where in the usual notation

$$\omega = 2\pi f, \qquad (f \text{ is the frequency}),$$

and

$$j = \sqrt{-1}.$$

In the transition layer ($0<x<h$) the solution of Equ. (2) is a linear combination of two transcendental polynomials

$$\left(1 + \frac{k-1}{h}\, x\right)^m \tag{5}$$

in which m corresponds to the two roots of the algebraic equation

$$m(m+1) + \frac{\omega^2 h^2}{(k-1)^2 v^2} = 0. \tag{6}$$

Since the elastic constants are continuous functions of x, both

$$u \text{ and } \frac{\partial u}{\partial x}$$

must be continuous for $x=0$ and $x=h$. These boundary conditions give four equations which, together with Equs. (4) and (5), lead to the reflection coefficient

$$R = \frac{1}{2\sigma + 2\gamma \coth(\gamma \log k)}, \tag{7}$$

where

$$\sigma = \frac{j\omega h}{(k-1)v},$$

and

$$\gamma = \sqrt{\tfrac{1}{4} + \sigma^2}.$$

In general R is complex, so that the phase as well as the amplitude of the reflected wave is a function of frequency. For low values of frequency ($|\sigma| \ll 1$), Equ. (7) reduces to Equ. (1), as in the absence of the transition layer. As the frequency increases from zero, R decreases slowly at first and then more rapidly, until it becomes zero when

$$|\gamma| \log k = \pi.$$

This point corresponds to a frequency given by

$$f_0 = \frac{v}{2h} \frac{k-1}{\log k} \left[1 + \left(\frac{\log k}{2\pi}\right)^2\right]^{1/2}. \tag{8}$$

It is easy to see that, once the frequency f_0 is exceeded, R remains quite small. In a sense therefore, f_0 can be regarded as a cut-off frequency for reflections. In a specific case when $v = 8{,}000$ ft./sec., $h = 100$ ft., $k = 1.5$, the cut-off frequency is $f_0 = 40$ cycles/sec. Fig. 2 gives the value of R as a function of f/f_0 for $k = 1.2,\ 1.5,\ 2.0$.

2. THE REFLECTION AT LARGE ANGLES OF INCIDENCE

As stated before, the exact treatment of this problem is very complicated because of the transformation of longitudinal waves into transverse. Even though the latter be neglected, the coefficient of reflection is still given by a very complicated expression involving Bessel functions of complex order. It does not lend itself to calculation, and it seems quite useless to reproduce it here.

Nevertheless, it is possible to give a broad outline of the manner of variation of the reflection coefficient with the angle of incidence, without going through any detailed analysis. First, consider waves of frequency so high that geometrical optics gives a good enough description of their propagation. Such waves will not be reflected in the ordinary sense of the word. If, however, the angle of incidence exceeds a certain critical value, the waves which for the sake of definiteness may be supposed to originate in region A (Fig. 1), will be *refracted* back into A by the transition layer. From the point of view of an observer in A this is of course equivalent to total reflection. The critical angle i is given by the equation

$$\sin i = 1/k. \tag{9}$$

The coefficient of reflection for short wave lengths is therefore zero until the angle i is reached, and then suddenly jumps to the value 1.

If the wave length is great compared to the thickness of the transition layer, the latter can be disregarded, and the theory of reflection from discontinuties can be used. It seems clear that for intermediate wave lengths the reflection coefficient will be between these two extremes.

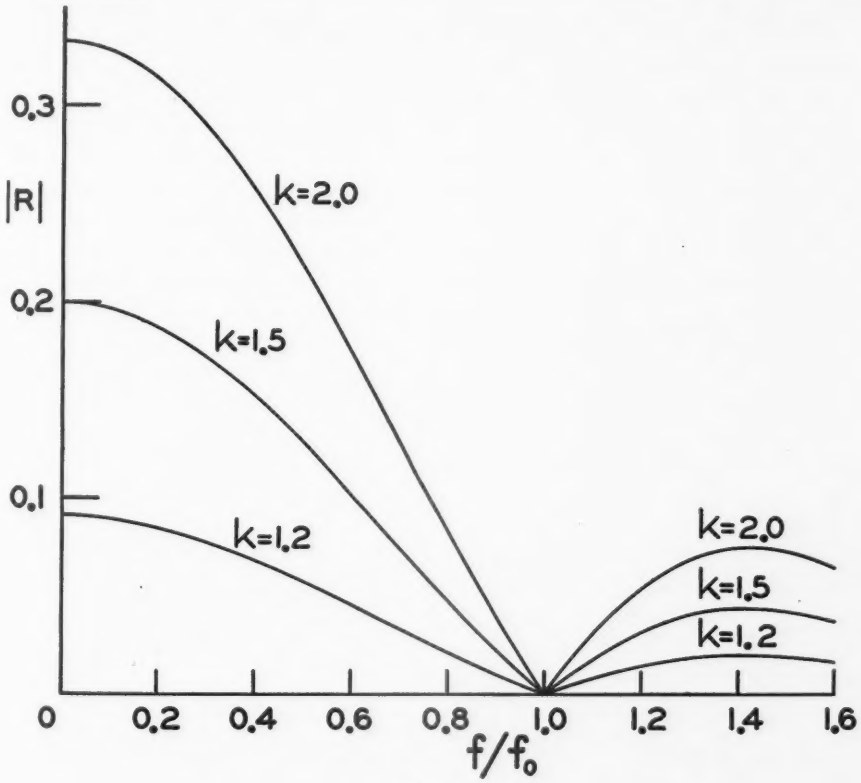

FIG. 2

3. THE REFLECTION OF PULSES

The discussion must be limited to vertical incidence. A pulse of arbitrary shape can be represented as a Fourier integral of harmonic waves. The reflection of any of these component waves is described by Equ. (7), and by combining them it is possible to obtain the shape and the amplitude of any arbitrary pulse. This is standard procedure in such problems.

First of all, in order to be able to handle the resulting expressions, it is necessary to simplify Equ. (7). Fortunately, when the velocity ratio k has a moderate value (e.g. when $k<2$), it is possible to neglect the difference between γ and σ. Equ. (7) can then be written as

$$R = \frac{\sin(|\sigma| \log k)}{2|\sigma|} \exp(-\sigma \log k). \tag{10}$$

Let the incident wave consist of a rectangular pulse of infinitesimal duration T_0 and amplitude E (Fig. 3a). Such a pulse can be expressed formally as the Fourier integral

$$\frac{ET_0}{\pi} \int_0^\infty \cos \omega t \cdot d\omega. \tag{11}$$

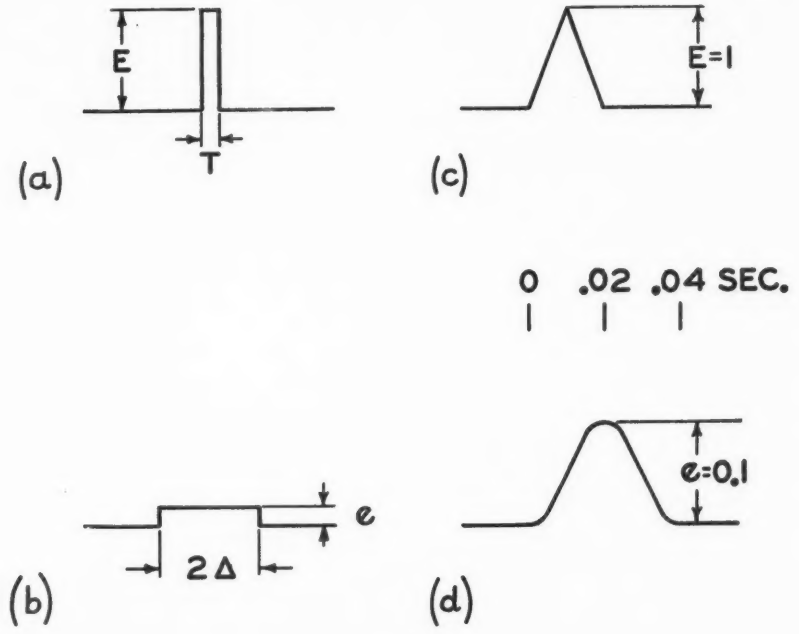

FIG. 3

The Fourier integral which represents the reflected wave, obtained by the application of Equ. (10) to Equ. (11), becomes

$$e = \frac{ET_0}{\pi} \frac{\log k}{4\Delta} \int_0^\infty 2 \frac{\sin \omega\Delta}{\omega} \cos \omega(t - \Delta) \cdot d\omega, \tag{12}$$

where

$$\Delta = \frac{h \cdot \log k}{(k-1)v}$$

is the time necessary for the wave front to travel once across the transition layer. Equ. (11) corresponds to a rectangular pulse of finite duration 2Δ and the infinitesimal amplitude (Fig. 3b)

$$ET_0 \frac{\log k}{4\Delta}.$$

By employing these results and the principle of superposition, it is possible to determine the reflection of a wave of any shape since it can always be regarded as a simple sum of such rectangular pulses. As an example, the incident wave in Fig. 3c is transformed into the reflection shown in Fig. 3d. This case was calculated on the assumption that the reflecting transition layer is 100 ft. thick, and separates two regions in which the velocities are respectively 8,000 ft./sec. and 12,000 ft./sec. The time $\Delta = 0.01$ sec. The amplitude of the reflection is 0.1 the amplitude of the original pulse, whereas the coefficient of reflection of long waves under the same conditions would be 0.2.

RECENT PAPERS OF INTEREST TO EXPLORATION GEOPHYSICISTS

The following papers appear in the recent edition of the Transactions of the American Geophysical Union, published by the National Research Council.

On the estimation of temperatures at moderate depths in the crust of the earth. C. E. Van Orstrand.

The external gravity-field and the interior of the earth. W. D. Lambert.

The earth's interior as inferred from terrestrial magnetism. A. G. McNish.

Gravity-measurements on the U.S.S. *Barracuda*. Maurice Ewing.

Geological interpretation of data collected on cruise of U.S.S. *Barracuda* in the West Indies—preliminary report. H. H. Hess.

Crystal-chronometer time in gravity-surveys. Albert J. Hoskinson.

Recent gravity-work of the United States Coast and Geodetic Survey. Carl I. Aslakson.

Geodetic work executed in the Republic of Mexico in 1936. Manuel Medina.

Gravity-anomalies and geologic structure. George P. Woollard.

Accuracy of modern gravimeter-measurements. Donald C. Barton and W. T. White.

Current geophysical activity in Texas, Louisiana, and the Mid-continent. Donald C. Barton.

Seismic refraction methods as applied to shallow subsurface exploration. E. R. Shepard.

Progress-report in seismology for the United States Coast and Geodetic Survey and cooperating stations. E. W. Eickelberg.

Timiskaming earthquake-data and time-distance curves for dilatational waves. Ernest A. Hodgson.

A low-cost seismograph and recording drum. H. Rutherford and R. L. Alkire.

The torsion-pendulum analyser as a double integrator. Frank Neumann.

Earth-current variation with periods longer than one day. W. J. Rooney.

An astatic magnetometer for measuring the susceptibility of materials for magnetic instruments. E. A. Johnson and W. F. Steiner.

A zero distribution-coefficient for horizontal-intensity magnetometers. Geo. Hartnell.

World-wide changes in potential gradient. G. R. Wait and John W. Mauchly.

Magnetic work of the United States Coast and Geodetic Survey, April 1936 to March 1937. R. S. Patton.

Report on research in terrestrial magnetism by the Department of Geology of the University of North Carolina. W. F. Prouty.

Magnetic work of the Dominion Observatory, Ottawa, during 1936. R. Meldrum Stewart.

Progress-report on researches in terrestrial magnetism and electricity at Department of Terrestrial Magnetism, Carnegie Institution of Washington, for year April 1936 to March 1937. J. A. Fleming.

The value of geophysical methods in ground-water studies. O. E. Meinzer.

Resistivity-studies of some salt-water boundaries in the Hawaiian Islands. J. H. Swartz.

The use of resistivity-methods in the location of salt-water bodies in El Paso, Texas, area. A. N. Sayre and E. L. Stephenson.

Results to be expected from resistivity-measurements. B. E. Jones.

Search for ground-waters by the electrical resistivity-method. L. E. Workman and M. M. Leighton.

Prospecting for water with geophysical methods. C. A. Heiland.

The following papers appear in Bd. 7, Heft 1 of the Beiträge zur Angewandten Geophysik.

Erfahrungen mit dem seismischen Refraktionsverfahren bei der geophysikalischen Reichsaufnahme. H. Reich.

Seismische Bestimmung einer reflektierenden Ebene. F. Gassmann.

Zur Analyse von Schwereanomalien. E. A. Ansel.

Some graphical solutions in torsion balance prospecting. V. Gavrilovich Gabriel.

Überwachung von Tiefbohrungen. C. Mainka.

Beiträge zur Funkgeologie. V. Einige Untersuchungen uber die Funkmutung nach dem Sog. Kapazitätsverfahren. Volker Fritsch.

Geothermische Messungen in Bergwerken und Übersicht über die Ergebnisse der Geothermik. J. G. Koenigsberger.

THE SOCIETY ROUND TABLE

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The Executive Committee have approved for publication the names of the following candidates for membership in the Society. This publication does not constitute an election, but places the names before the membership at large. If any member has information bearing on the qualification of these nominees, he should send it to the Secretary within thirty days. (Names of sponsors are placed beneath the name of each nominee.)

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Volume II Number 4

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OCTOBER, 1937

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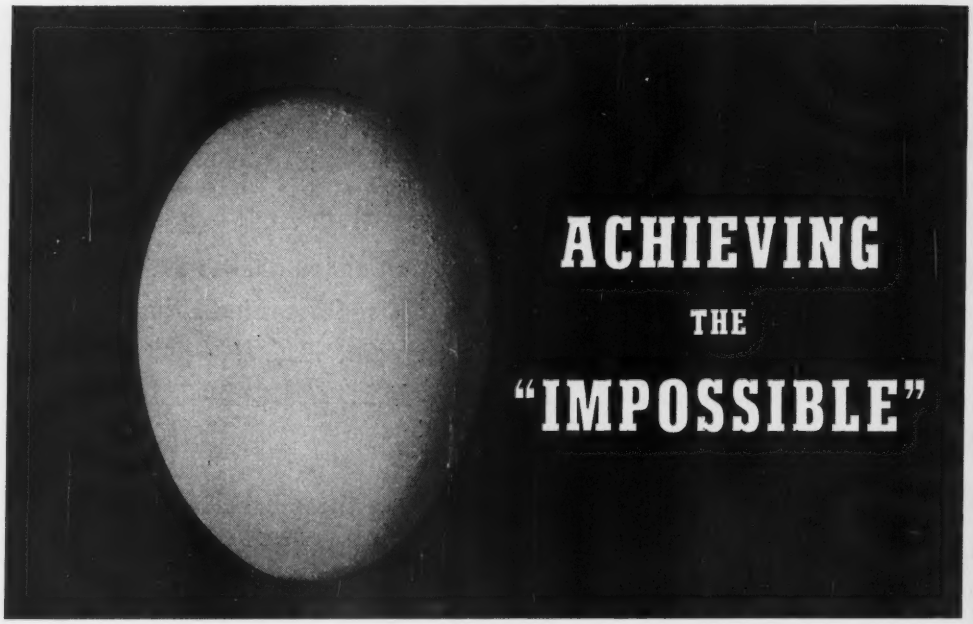

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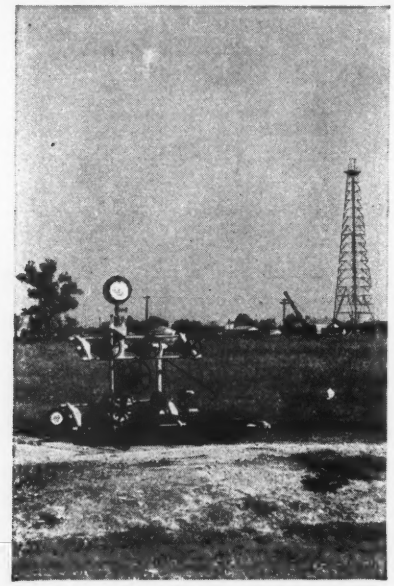

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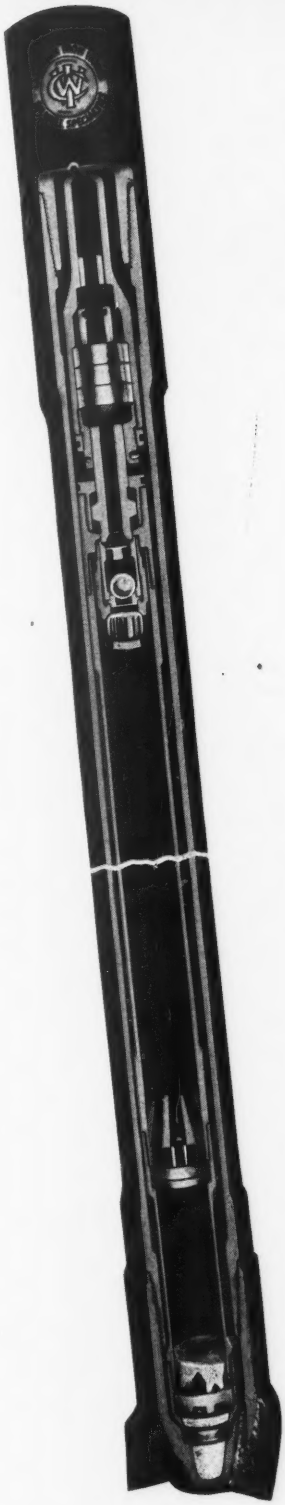

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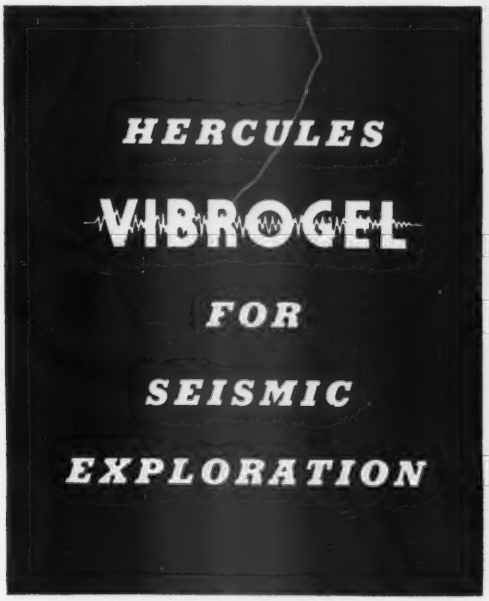

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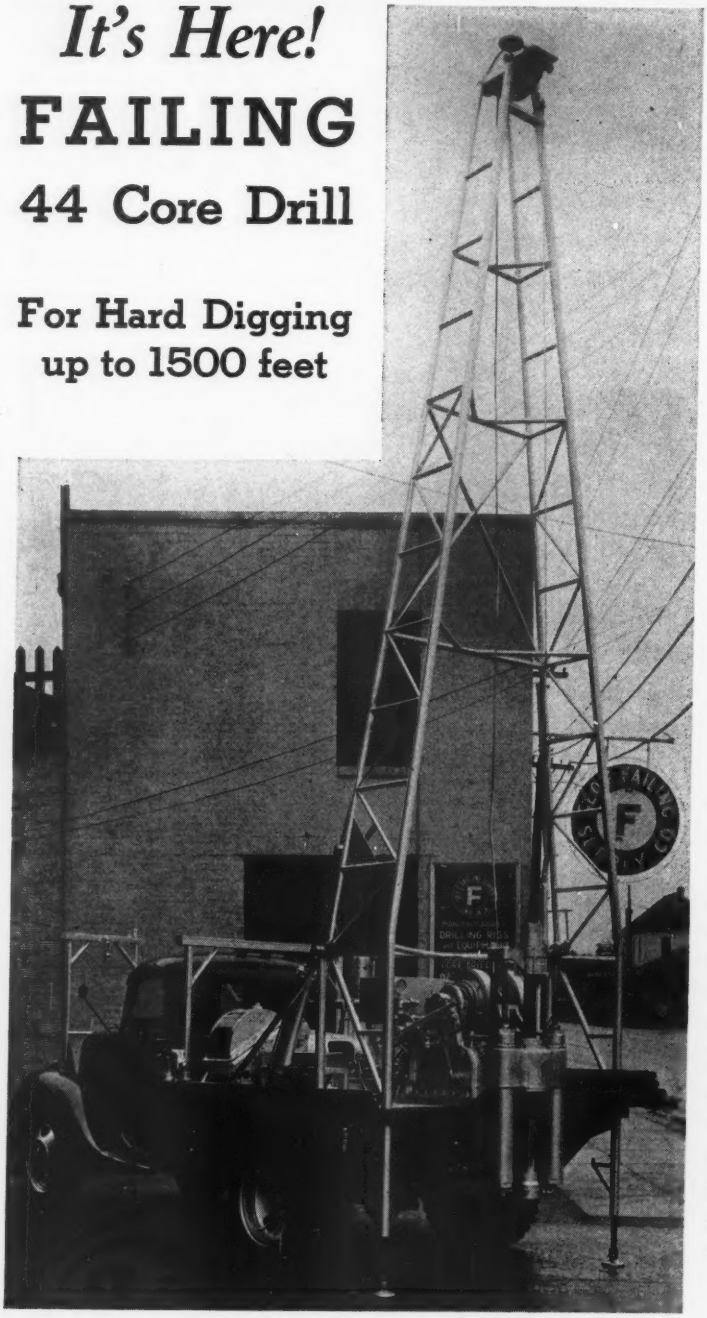

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